DOCUMENT RESUME

ED 021 754 SE 004 948

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SCHOOL MATHEMATICS STUDY GROUP REPORT NO. 6, PRELIMINARY REPORT ON AN EXPERIMENT WITH JUNIOR HIGH SCHOOL VERY LOW ACHIEVERS IN MATHEMATICS.

Stanford Univ., Calif. School Mathematics Study Group.

Spons Agency-National Science Foundation, Washington, D.C.

Pub Date 68

Note-117p.

EDRS Price MF-$0.50 HC-$4.76

Descriptors-ACADEMIC ACHIEVEMENT, *ACHIEVEMENT, *CURRICULUM, CURRICULUM DEVELOPMENT, INSTRUCTION, LEARNING, LOW ABILITY STUDENTS, *LOW ACHIEVERS, *MATHEMATICS, *SECONDARY SCHOOL MATHEMATICS

Identifiers-School Mathematics Study Group

This report presents the initial findings of a study of low achievers in junior high school mathematics. The regular syllabus for seventh grade mathematics in the school dictated the mathematical topics to be attempted, and a series of units was developed and tried out during the year. Underlying the instruction was the primary objective that the pupils have successful computational experiences. With the exception of standardized testing, students were allowed to use their notes and tables under all conditions. The analysis of data obtained from an experiment involving an experimental group and a control group indicated that both groups made substantial gains, with the control group making greater gains than the experimental group. The appendix to this manual contains a table of contents which outlines the material covered by the pupils in the experimental group. Samples of work sheets and tables are included. (RP)

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SMSG REPORTS

No. 6

Preliminary Report on an Experiment with Junior High School Very Low Achievers in Mathematics

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I INTRODUCTION

This report presents the initial findings of a study of junior high school students underachieving in mathematics. This study resulted from a Conference on Mathematics Education for Below Average Achievers sponsored by SMSG in Chicago, Illinois, on April 10 and 11, 1964, with financial support from the Cooperative Research Branch of the U. S. Office of Education. Position papers were presented for discussion at the conference, and the participants were asked to react to the papers and to the discussions. The participants then met in four groups, each devoted to a particular area of concern. These areas were:

Schools in the slum areas of the great cities;
Segregated Negro schools;
Mathematics for the unemployed;
Mathematics programs for students of low ability.

Recommendations of the four groups were discussed at a plenary session, circulated to all participants for comments, and incorporated in the published report of the conference.[1]

An ad hoc SMSG committee met in May of 1964 to review the recommendation from the conference. This committee urged SMSG to undertake certain specific activities, one of which was the preparation of experimental materials:

> "For junior high school students (and perhaps others): Materials which relieve the students from the burdens of computation as much as possible, by providing slide rules, books of tables, pocket computers, etc."

The reason for this recommendation was the conjecture that many underachieving junior high school students had experienced failure in elementary school mathematics, had been forced to do extensive drill in computation, had continued to fail, and that continued failure had led to intense dislike and even fear of computation, and in turn, to an expectation of continued failure.

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[1] School Mathematics Study Group, Conference on Mathematics Education for Below Average Achievers, Stanford, 1964.

II EXPLORATORY STUDY

As a first step, an exploratory study was conducted during the 1965-66 academic year. The cooperation of a school district in Santa Clara County, California, was obtained to permit this investigator to teach a class of selected seventh grade students. This class was located in a community which might be classified as a "bedroom" area, with most residents of middle or low-middle socio-economic status.

III PUPIL SELECTION

For purposes of seventh grade homogeneous grouping, this district, in the spring of each year, administers the California Achievement Tests, in reading and arithmetic, to all sixth grade pupils. The results of these tests were used as a preliminary screening instrument for the selection of pupils for the proposed class.

In selecting pupils for this class, an attempt was made to eliminate any pupil whose record indicated obvious social or psychological problems, chronic absenteeism or truancy, or who could present a discipline problem. In addition, it was felt desirable that these pupils be of average intelligence and be working at grade level in all subjects but mathematics, in which case they were to be one year or more below grade level.

Interviews were obtained with the principals of the four feeder elementary schools and, when necessary, with the sixth grade teachers. Cumulation folders were also inspected. As a result of this investigation, from an original list of fifty-one possible candidates, twenty-one pupils were selected and programmed into this special class for the school year 1965-66.

TABLE 1

CALIFORNIA ACHIEVEMENT TEST

March, 1965. Actual Grade Placement, 6.6

	Mean Grade Placement	Range
Arithmetic Reasoning	5.56	1.8
Arithmetic Fundamentals	5.67	2.2

Table 1 shows the mean grade placement of the pupils selected for the special class to be about one year below grade level in both arithmetic reasoning and in arithmetic fundamentals.

I.Q. scores were taken from the pupils' cumulation folders. In most cases, I.Q. scores were determined by the California Test of Mental Maturity. Mean I.Q. was 100, with a range of 55.

IV PUPIL BEHAVIOR AND ATTITUDES

Initially, certain behavior and attitude patterns were observed which were characteristic of the pupils. Generally:

a. There was a severe lack of organization in their approach to learning.

b. They appeared to be immature when compared to other seventh grade pupils.

c. Their attention span was exceptionally short.

d. Their interest span was exceptionally short.

e. Negative attitudes existed toward mathematics, and in some cases, toward school in general.

f. Pupils exhibited a defeatist attitude with regard to their ability to succeed in mathematics.

V PARENT CONFERENCES

Conferences were held with the parents of nineteen of the twenty-one pupils (the parents of two of the pupils were never able to attend). Results of these parent conferences revealed the following:

a. Parents were unaware their child was below grade level, but all realized their child was having difficulty in mathematics.

b. There was a tendency to blame some particular grade teacher for their child's deficiency.

c. Parents indicated the end of the third grade or the beginning of the fourth grade as the point where most of the difficulty began. Much of the trouble appeared to have started with the multiplication tables and was then compounded by the child's inability to do division.

Remedial work with all of these pupils seemed to have been based on drill. There was no indication that any alternative processes to any of the standard algorisms were used.

VI PROGRAM DEVELOPMENT

At its conception, it was agreed that the approach to the development of this program would be a pragmatic one, in that the program would result from the investigator's daily experiences with this exploratory class.

Rather than predetermining the mathematical content necessary for these pupils and then constructing a sequential program and materials accordingly, various concepts and processes would be presented daily, and, if successful, pursued as far as the capabilities of the pupils allowed. On the other hand, if the presentation of a concept of process should prove to be beyond the students' capabilities or did not appear to be the best approach, it would be either discarded completely or delayed until a later date. This technique was used in the development of most of the materials which were used in the study.

The regular syllabus for seventh grade mathematics in the school dictated the mathematical topics to be attempted, and a series of units was developed and tried out during the year.

Underlying the instruction was the primary objective that these pupils have successful computational experiences. Various tables were issued and pupils were urged to use them. With the exception of standardized testing, students were allowed to use their notes and tables under all conditions.

Originally, it was the intent that computational devices other than tables be introduced into the classroom. But, as the program progressed, and it became apparent that learning was taking place, it was this investigator's decision not to include these other devices.

A. Posttest

TABLE 2

CALIFORNIA ACHIEVEMENT TEST

March, 1966. Actual Grade Placement, 7.6

	1965	range	1966	range	GAIN
A. R.	5.56	2.1	8.15	4.0	2.59
A. F.	5.67	1.9	7.02	1.8	1.35

Table 2 shows the arithmetic grade placement scores on the pretests which were given to the students in March, prior to the exploratory period, and on the posttest which was administered one year later. The group means show that approximately 2.6 years were gained in Arithmetic Reasoning and 1.4 years in Arithmetic Fundamentals. In relation to their actual grade placement, the class mean in Arithmetic Reasoning was approximately 0.6 year above grade level and in Arithmetic Fundamentals, now only about 0.6 year below grade level.

B. 1966-67

This investigator continued teaching this class, now in grade eight, during the 1966-67 academic year.

The original class consisted of twenty-one pupils. During the summer of 1966, three pupils moved from the district. A fourth pupil was removed from the class and placed in special classes. For reasons of scheduling, three additional pupils were placed in the class. These pupils were not included in any testing.

C. Procedures

Procedures during the better part of the second year of this study followed closely those described as being employed during the first year.

When possible, introduction of any topic included the reteaching and reenforcement of those concepts and processes introduced the previous year.

By March of 1967, those topics in junior high school mathematics normally covered by this date had been covered by this class.

D. Posttest - 1967

TABLE 3

CALIFORNIA ACHIEVEMENT TEST

March, 1967. Actual Grade Placement, 8.6

	1965	1966	GAIN	1967	GAIN	GAIN (2 years)
A. R.	5.7	8.3	2.6	9.5	1.2	3.8
A. F.	5.6	7.1	1.5	8.8	1.7	3.2

Table 3 shows the grade placement means over the two-year testing period.[2] The group means show that for the second year of the study, 1.2 years were gained in Arithmetic Reasoning, for an overall gain of 3.8 years. In Arithmetic Fundamentals, gain for the second year was 1.7 years, for an overall gain of 3.2 years. In relation to the pupils' actual grade placement, the class mean in Arithmetic Reasoning shows them to be 0.9 year above grade level, and in Arithmetic Fundamentals, 0.2 year above grade level.

It is interesting to note that the greatest gain in Arithmetic Reasoning took place the first year, with a tendency to level off the second year. On the other hand, Arithmetic Fundamentals show a consistent gain, with a tendency to increase from one year to the next.

E. Main Study - 1966-67

The encouraging results of the pilot study in the 1965-66 academic year led to a decision to try the material and methods developed during that year with a larger number of classes. Ten school[3] in the area south of San Francisco agreed to contribute one experimental seventh grade class each, and five[4] others allowed a seventh grade class, designated as a control class, to be tested at the beginning and end of the school year.

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[2] The discrepancy between the entry in Table 2 under Gain in Arithmetic Fundamentals and that entered in Table 3 is accounted for by the four students no longer in the study. In computing the means, Table 2 includes these pupils; Table 3 does not.

[3] A list of the participating school districts, junior high schools, and teachers appears in Appendix 1 on page 25.

F. Description of the Schools

The schools participating in this seventh grade pilot study were chosen on the basis of consistency in socio-economic setting and, for the purpose of seventh grade placement, consistency in testing procedures.

The ten participating schools are located in communities that can be classified as "bedroom areas" of middle to low-middle class. There do exist, within these areas, pockets of both low and high-middle class neighborhoods. There are no areas that can be classified as slums. Selection of these particular schools eliminated any problem which might stem from the bilingual student.

In nine of these schools, the California Achievement Test, in both reading and arithmetic, is administered to all sixth grade pupils in the spring of each year. The results of these tests are then used by the junior high school counselor for placement of students in the seventh grade. In the one exception, testing takes place in the spring of the fifth grade.

From these placement tests, this investigator compiled the initial list of possible students to participate in the pilot study.

G. Description of the Pupil - Pilot Classes

The criteria for selection of pupils for the pilot classes were the same as those used in the exploratory study.[5] Every effort was made to eliminate any pupil with a behavioral problem.

Interviews were obtained with the elementary school principal, and if necessary, with the pupil's sixth grade teacher. Cumulation folders were also inspected. Those pupils who deviated greatly from the selection criteria were eliminated from the study.

Eight of the participating schools reported results of the CAT Arithmetic Tests in terms of grade placement in Total Arithmetic. Two schools reported results in percentile ranking.

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[4] Originally, there were seven control classes, but it proved impossible to test two of them in the spring.

[5] See page 2 of this report.

It was found that, of the 261 pupils participating initially in the pilot study:

a. Eight classes (204 pupils) had a mean Total Arithmetic grade placement 1.0 year below grade level.

b. One class (27 pupils) had a mean Total Arithmetic score which placed them in the twentieth percentile.

c. One class (30 pupils) had a Total Arithmetic score which placed them in the forty-fifth percentile.

H. Selection of the Teacher - Pilot Classes

Selection of the teachers for the pilot classes was based on the following criteria:

a. Three to six years teaching experience, preferably, but not necessarily, in mathematics.

b. An ability to identify with the underachieving pupil without being overly sentimental.

c. A willingness to experiment with, and employ, methods of curriculum and instruction that might deviate radically from the accepted structural presentation of seventh grade mathematics.

d. A willingness to attend in-service seminars, complete required reports, and be observed in the classroom under teaching conditions.

e. An undergraduate major and minor in areas other than mathematics.

Principals of the ten participating schools were interviewed with regard to staff members that would meet, as closely as possible, the selection criteria. Each principal recommended one teacher he felt fit the criteria. The philosophy, structure, goals, and requirements of the program were explained by this investigator and the decision whether to participate or not left to the discretion of the teacher. In each case, the recommended teacher was willing to participate in the program.

I. Teacher Background - Pilot Classes

Background information on the participating teachers was obtained. Although there was a wide range in many of the categories of teacher background, the following are the central tendencies in some of the more pertinent areas.

Most of the teachers had been teaching six years, with a B.A. as their highest degree. They had acquired in this time in excess of 30 academic credits beyond the B.A. most had taken no credits in college mathematics at the level of calculus or beyond, but had taken 4 to 6 credits in methods of teaching mathematics. All of them had involved themselves in other types of preparation in mathematics.

Only one teacher held an administrative credential. None were working toward an administrative credential, held a counseling credential, or were working toward a counseling credential.

Undergraduate majors are as follows:

a. Five in Elementary Education

b. Three in Social Science

c. One in Physical Education

d. One in Art

Undergraduate minors are as follows:

a. Three in English

b. Three in Science

c. Two in Social Science

d. One in Business

e. One in Physical Education

J. Control Classes

The pupils and teachers who acted as control classes for this study could not be chosen as selectively as those in the pilot study. This condition was due to the volunteer basis under which the control classes, and their teachers, participated in the study.

Seven classes, in schools different from those containing the pilot classes, volunteered to act as control for this study. Selection criteria, for both pupil and teacher, were explained and the choice left to the discretion of the school's counselor.

K. In-Service Seminars

During the summer of 1966, teachers of the pilot classes were provided with the SMSG, Introduction to Secondary School Mathematics, Volumes 1 and 2, and the accompanying commentaries. In addition, Studies In Mathematics, Volume IX, A Brief Course in Mathematics for Elementary School Teachers, was supplied each teacher. These volumes were to be used for reference concerning any of the topics contained in the student material.

At the beginning of the school year, seminars with the teachers of the pilot classes were conducted at two week intervals. As the program progressed, and problems involving distribution of material, classroom procedures, and other administrative details were alleviated, meetings were then limited to once a month. Between such meetings, this experimenter contacted each pilot teacher and acted in an advisory capacity. No contact was made with the teachers of the control classes.

Discussions during these seminars centered on methods of presentation of various subject matter topics and the problems encountered by individual teachers.

L. Description of the Material

During the summer of 1966, the work sheets used in the exploratory study were revised and extended. A total of 22 units were prepared. Development of these units was based primarily on those successful experiences encountered by the experimenter while teaching the exploratory class.

To a limited degree, each unit was developed to be independent of any other unit. It was found that with the short attention span of these pupils, greater participation would result if any one topic was not dwelt upon for any great length of time. Constructing the units in this manner would then allow the teacher greater flexibility in selecting topics that were of interest to the pupils.

Each unit consists of a number of daily work sheets.[6] The lessons on these work sheets were constructed to be short and complete within themselves, with many examples for the pupil to follow. If, because of the nature of the topic, a lesson required a more lengthy explanation, the lesson was then partially programmed. With the attention span of these pupils being as short as it was, this approach put the pupil quickly to work, allowed ample time for supervised study, and required little or no homework.

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6. Sample work sheets may be found in Appendix 2.

The work sheets were handed out daily and contained ample space to do any of the required work. Work sheets were placed in a binder which, in most cases, was kept in the classroom. There were two factors that prompted this approach. First, because of these pupils' apparent lack of organization, a text book became a handicap to them. Simply keeping track of its physical location appeared to be beyond the capabilities of most of the pupils. Secondly, this approach seemed to produce the more positive effect of having the pupils consider what they had accomplished, rather than projecting what they had yet to do.

In the content of these units, little reference is made to any practical application or social utilization of mathematics. It had appeared to this investigator in the pilot study that the pupils were more highly motivated by successful performance and by the ability to see mathematical relationships fall into patterns than by being able to apply that mathematics they were learning to some practical situation.

M. Pupil Testing Program

A battery of tests was administered at the start of the school year to students in both pilot and control classes. The classroom teacher administered the tests.

Stanford Achievement Test[7], Intermediate II, form W, using Test 1, Arithmetic Computation, and Test 3, Arithmetic Applications, was used as the standardized pretest. In addition, SMSG constructed tests on attitude (Opinion Inventory, form SC) and achievement (Mathematics Inventory, form SC) were administered.[8]

The following information was also gathered: sex, I.Q. of each student, the number of days absent during the school year, and a reading score (experimental students only). The number of units covered in each class was also recorded.

Brief evaluations and comments were requested from the ten experimental teachers and obtained from them. These appear in Appendix 3.

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[7] The entries in all tables under Stanford Achievement Test are reported in terms of grade placement. All entries in the SMSG tables are raw scores.

[8] These tests, which are described in Appendix 4, will be reproduced in an SMSG technical report early in 1968.

For posttesting purposes, Stanford Achievement Test, Intermediate II, form X, was administered at the end of April. At the end of May, the SMSG tests were re-administered.

VII ANALYSIS OF THE 1966-67 DATA

Statistics on the fall tests are shown in Appendix 5 for each of the ten experimental classes and five control classes. Inspection of these tables does not indicate any major differences between classes. Because of the small number of classes involved, all the students in the experimental classes were pooled into one group and those in the control classes into a second group for all the statistical analyses which follow.

Mean scores for these two groups on the initial measures appear in Appendix 6. The mean number of units covered during the year and the mean number of days absent are also recorded.

None of the differences between the two groups were significant at the .05 level except for ATT 3 and units covered, for both of which the difference is significant at the .01 level. The calculations are shown in Appendix 7.

A. Gains on the Stanford Achievement Test Scales

Table 4 shows the mean scores for both groups for the spring and the fall administration of the two SAT scales.

TABLE 4

FALL AND SPRING SCORES - SAT SCALES [9]

Experimental Group

	Fall		Spring		
	Mean	S. D.	Mean	S. D.	Gain
Comp.	46.11	10.18	58.07	13.54	11.96
Appl.	53.82	13.27	58.99	13.41	5.17

Control Group

	Fall		Spring		
	Mean	S. D.	Mean	S. D.	Gain
Comp.	46.49	9.83	63.87	13.66	17.38
Appl.	54.63	11.73	61.92	13.77	7.29

It will be observed that both groups made substantial gains on both scales, with the control group making greater gains than the experimental group.

It was surmised that the posttest scores would depend not only on the pretest scores but also on I.Q., the number of days absent, and the number of units covered, and hence that it would be appropriate to adjust the posttest scores for differences in the pretest scores. Analysis of covariance was therefore attempted. For this, regression equations of the posttest scores were computed for the two groups separately for both the

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9. In writing the computational program for the analysis of the data, the SAT scale scores were multiplied by 10. Recall that these scores indicate grade placement.

Computation and Application scales. For both scales the regression planes turned out to be significantly non-parallel, at the .01 level for Computation and at the .05 level for Application. The calculations are shown in Appendix 8. This meant, of course, that analysis of covariance was not appropriate. More important, it meant that the experimental and control programs were significantly different and that the amount of learning in one program depended on the initial conditions in a different way from that in the other program.

B. Gains on the SMSG Scales

Table 5 shows the mean scores, for both groups, for the spring and the fall administrations of the five SMSG scales.

TABLE 5

FALL AND SPRING SCORES - SMSG SCALES

Experimental Group

	Fall		Spring		
	Mean	S. D.	Mean	S. D.	Gain
SMSG - 1	2.07	1.58	3.13	1.55	1.06
SMSG - 2	3.40	1.81	4.42	1.91	1.02
SMSG - 3	1.71	1.32	2.39	1.79	0.68
SMSG - 4	3.60	1.76	5.08	2.10	1.48
SMSG - 5	0.73	0.84	1.03	0.93	0.30

Control Group

	Fall		Spring		
	Mean	S. D.	Mean	S. D.	Gain
SMSG - 1	2.28	1.49	2.35	1.70	0.07
SMSG - 2	3.40	2.05	3.83	2.28	0.43
SMSG - 3	1.75	1.41	2.67	1.89	0.42
SMSG - 4	3.56	1.70	4.81	2.47	1.25
SMSG - 5	0.71	0.85	1.09	0.97	0.38

In all but one case, there were substantial gains from fall to spring for both groups. Analysis of covariance was again attempted in order to compare the gains of the two groups. Regressions of the posttest scores on the pretest scores and on I.Q., Units Covered, and Days Absent were computed. In no case did the regression planes for the two groups significantly differ from parallelism. Analysis of covariance was therefore carried out and indicated that the experimental groups had a significantly greater ($p < .05$) gain on scales 1, 2, and 4. The differences on scales 3 and 5 were not significant. Table 6 shows the adjusted means for the two groups. The computations are shown in Appendix 9.

TABLE 6

ADJUSTED MEANS - SMSG SCALES

	Experimental	Control
SMSG - 1	3.22	2.13
SMSG - 2	4.55	3.53
SMSG - 3	2.53	2.37
SMSG - 4	5.20	4.54
SMSG - 5	1.08	0.98

C. Changes on the Attitude Scales

Table 7 shows the mean scores, for both groups, for the spring and fall administrations of the attitude scales.

TABLE 7

FALL AND SPRING ATTITUDE SCALE SCORES

	Experimental Group				Control Group			
	Fall	Spring	Change	t[10]	Fall	Spring	Change	t[10]
ATT 1	20.39	20.37	-0.02	-0.04	19.91	20.73	0.82	1.11
ATT 2	13.17	14.07	0.90	2.55*	13.09	13.19	0.10	0.17
ATT 3	33.56	33.72	0.16	0.45	31.76	32.47	0.71	0.68
ATT 4	24.68	27.78	3.10	5.73**	24.31	25.78	1.47	1.52
ATT 5	35.51	30.22	-5.29	-8.49**	34.48	31.18	-3.30	-3.25**
ATT 6	24.77	24.51	-0.26	-0.52	24.68	24.78	0.10	0.13
ATT 7	29.79	26.99	-2.80	-4.68**	28.99	26.74	-2.25	-2.32*
ATT 8	27.85	31.64	3.79	6.51**	28.53	29.13	0.60	0.40

The larger number of significant changes for the experimental group is another indication that the experimental program does in fact differ, in its effects on students, from the standard program for low achieving seventh grade students.

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[10] Two tailed t for correlated data.

*Significant at the .05 level.

**Significant at the .01 level.

D. Reading

Since reading ability is one of the factors often taken into account in assigning students to ability groups or special programs in mathematics, and since reading scores had been obtained for the experimental students in this study, an analysis of the relationship of reading ability to mathematics achievement was undertaken.

For each of the five scales, SAT Computation, SAT Application, SMSG-1, SMSG-2, and SMSG-4, the regression of the posttest score on all the pretest measures and on Units Covered and Days Absent was computed by means of a stepwise procedure. This procedure first selects that independent variable which best predicts the posttest score in the sense of accounting for the largest part of the variance of the posttest score. The procedure next selects that one of the remaining independent variables which accounts for the largest part of the remaining variance after the effect of the first independent variable has been partialled out, and so on, until none of the remaining variables accounts for any significant amount of the remaining variances.

The results are shown in Appendix 10, which summarizes, for each of the five posttest scales, the order in which the dependent variables were chosen and, in column 5, the additional amount of variance accounted for at each step.

These summaries indicate that, when other variables are taken into account, reading accounts for a relatively small part of the total variance.

VIII DISCUSSION

Appendix 6 shows the mean grade placement of the pupils in the Experimental group to be 2.5 years below grade level in Computation and 1.7 years below grade level in Applications. These are substantially lower means than one would predict considering that the CAT tests, administered the previous spring, showed these pupils to be 1.0 years below grade level in Total Arithmetic.

In that the CAT tests were administered in the spring, whereas the SAT tests were administered the following September, regression over the summer could account for some of this difference. Also, the two tests may test different aptitudes.

From the viewpoint of the teacher though, the SAT tests came closer to indicating the actual mathematical performance level of the pupils in the

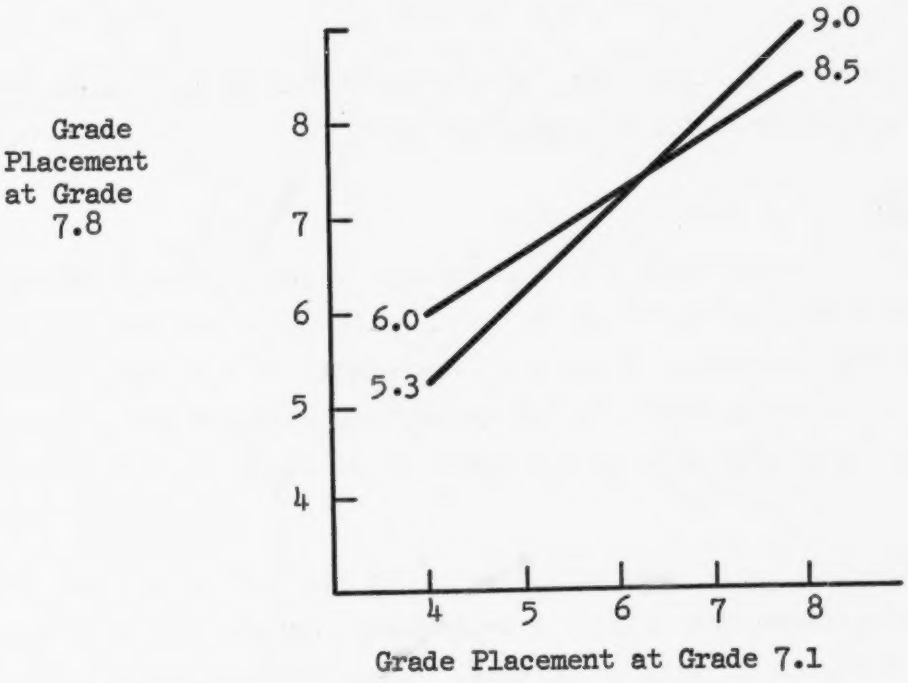
Experimental group. That is, the teachers felt that initially, the pupils in the Experimental group had mathematical skills closer to those expected of a fourth grade pupil.

It appears then, that teacher interpretation of standardized test scores can be an important factor when working with underachieving pupils. If, on standardized tests, discrepancies do exist that are as great as is indicated, this in itself could be a contributing factor to further underachievement. For, based on test scores, it would be possible to group pupils and place them in classes where the curricula being taught was far in advance of the achievement level of the pupils.

As can be seen in Appendix 6, the study started with rather tightly grouped classes of pupils. As the year progressed, the range in the pupils' computational skills began to spread. It appears that no provisions were made to meet the needs of those pupils who advanced more rapidly, and as a result the material used by the Experimental group may have limited the achievement potential of the more rapidly advancing pupil.

Figure 1, below, shows the regression (prediction) lines for both groups using pre-computation as a predictor of post-computation.

Figure 1



The slope of the regression line for the Experimental group is .92, which indicates that gain in computation was nearly constant regardless of the computational skills of the pupil at the beginning of the program. This consistency in gain across a wide range of initial computational skills seemingly supports the conjecture that the material used by the Experimental group may have had a limiting effect on the more rapidly advancing pupil.

Of particular interest is the point of intersection of the two regression lines. This point of intersection is at about 6.0 on the 7.1 grade placement axis. In terms of computational skills, this suggests that if a pupil is about 1 year below grade level, or above, his progress will be greater under the Experimental program. If, on the other hand, he is more than 1 year below grade level, progress in computation will be greater under a normal program. But, as is suggested by the result of the attitude scales, the methods generally employed to attain gain in computational skills under a normal program may be detrimental to the acquisition of positive attitudes toward mathematics in general.

Because of the underachiever's short interest and attention span, the material for the program was designed so that each unit would be short and, to a limited degree, independent of the preceding unit. This approach would allow for greater flexibility in the teaching of these pupils. Also, the spiraling of the content throughout the units permitted moving on to a new concept before the first one was mastered. Gains made by the pupils in the Experimental group indicate that this approach will work as successfully as programs where emphasis is placed on a rigid sequential approach.

There is a second statistic that can be interpreted as supporting, to a degree, the independence of topics in elementary school mathematics. A careful check was made on the attendance of the pupils in both the Experimental and Control groups. Approximately 13 percent (n = 30) of the pupils in the Experimental group were absent 16 or more class periods for a mean absence of 21.7 class periods. This is about 11 percent of the actual teaching time. Approximately 10 percent (n = 12) of the pupils in the Control group were absent 16 or more class periods for a mean absence of 21.6 class periods.

In scanning the abbreviated table below, it is apparent that the within group differences between the absentees and the non-absentees are not meaningful. In other words, those pupils that were absent so frequently experienced gain equivalent to those pupils who were more consistently in attendance.

ABSENTEES vs NON-ABSENTEES[11]

	Mean Gain Computation	Mean Gain Applications
Experimental Non-Absentees	1.1	0.4
Experimental Absentees	1.2	0.3
Control Non-Absentees	1.8	0.7
Control Absentees	2.1	0.5

These figures also suggest that chronic absenteeism may not impede the learning processes in elementary school mathematics to as great a degree as is generally believed.

An attempt was made, in the organization and exposition of the material in this program, to eliminate as much as possible complete reliance on the teacher for any success the program might have. That is, the function of the teacher was to be less that of the "lecturer" or "pedagogue" and more that of the "demonstrator", "tutor", and "supervisor" with little emphasis on complaisant observation and much emphasis on active pupil participation.

Within the Experimental group changes which took place were fairly constant across classes with relatively little variation. The same did not hold true for the Control group. There was little consistency and a great deal of variation between classes. This suggests that with the Control group, changes that occured were more closely associated with teacher influence, whereas with the Experimental group, changes were more closely related to the influence of the entire program.

Table 6 shows the gains made by both groups on the five SMSG Mathematics Inventory scales. It shows that the Control group made no significant gain on scale SMSG-1, which is composed of such basic properties as commutativity, associativity, distributivity, and inverse elements with respect to addition and multiplication of whole numbers. On the other hand, significant gain was made on the scale SMSG-4, which is composed of items involving either the notation of whole numbers or operations on them. This suggests that, with

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11. The complete table may be found in Appendix 11 on page 114.

the pupils in the Control classes, emphasis was placed on drill involving operations with whole numbers, with little emphasis on the understanding of basic structure.

As indicated in Table 7, the Experimental group experienced significantly higher adjusted gains on scales SMSG-1 (Whole Number Structure), SMSG-2 (Open Sentences - Operations), and SMSG-4 (Whole Numbers-1). This implies that, for the underachieving pupil, material that concentrates on mathematical structures, concepts, and relationships will do more to overcome the pupils' deficits in these areas than the standard remedial treatment of concentrating on drill.

Table 8 shows the mean scores, for both groups, for the spring and fall administrations of the attitude scales. In scanning this table, what is immediately apparent is that the pupils in the experimental group showed significant change on three scales which the Control group did not. Furthermore, on the two scales where both groups showed change, the Experimental group showed greater change than did the Control group.

Scale ATT 2 refelcts the pleasure or boredom a pupil experiences with regard to mathematics both in an absolute sense and comparatively with other subjects. The mean score on the pretest indicates that at the start of the program, pupils in the Experimental group did not know whether, for them, mathematics was fun or dull. The mean score on the posttest indicates that this noncommittal feeling had shifted closer to that of agreement, that mathematics was more pleasurable.

Scale ATT 4 reflects the ease or difficulty which a pupil associates with mathematics performance. Interpretation of these means indicates that, at the start of the program, the pupils more closely associated performance in mathematics with being difficult. Posttest means show a shift toward associating mathematics performance with being easy.

Scale ATT 5 reflects how a child wishes he were in his relationship to mathematics. The pretest mean on this scale indicates that at the start of the program there existed a strong desire, on the part of the pupils, to have a more positive relationship to mathematics. The posttest mean is significantly lower than at the start of the program. There are two interpretations that could account for this change. First, because the Experimental program was designed so that pupils would have successful experiences, it is possible that what initially was a desire on the pupils' part, ultimately became a reality. This would have the effect of producing a lower mean on the posttest. Secondly, it is also possible that the mathematical experience encountered by the pupils was such that they acquired a more realistic view of themselves and their relationship to mathematics. This could have the effect of lowering their ideals and, consequently, their performance on this scale.

Scale ATT 7 reflects the degree to which a pupil's mathematics performance is harmed by stressful conditions (e.g., examinations). The substantial difference between pretest and posttest suggests that the program has had the effect of relieving the pupils of some of the stressful conditions which, in the past, may have harmed their mathematical performance.

Scale ATT 8 reflects how a child sees himself in relation to mathematics. The mean pretest and posttest scores seem to indicate, that with the Experimental group, the program has had the effect of strengthening the pupils' actual self-concept of their ability to function mathematically.

As previously reported in describing the Exploratory study, little reference was made to any practical application or social utilization of the mathematical content of the pilot material. Pupils appeared to be more highly motivated by successful performance and by the ability to see mathematical relationships fall into patterns than by being able to apply that mathematics they were learning to some practical situation. On the other hand, the texts used by the pupils in the Control classes were laden with problem sets constructed to implement the content in a practical sense. The significant changes in attitude experienced by the Experimental group but not by the Control group suggest that the approach used in the Experimental material does more to motivate the underachieving pupil than the generally recommended approach of practical application.

That the Experimental group showed significant change on the three attitude scales ATT 2 (Math Fun vs Dull), ATT 4 (Math Easy vs Hard), and ATT 8 (Actual Math Self-Concept), whereas the Control group did not, indicates that the program developed for these underachieving pupils may possess certain qualities that bring about positive attitude change with respect to mathematics that does not occur when the underachiever is taught by the more traditional approach with the more conventional materials.

It would be reasonable to conclude that the first year of the Experimental study has resulted in successful mathematical growth for the participating pupils. In addition, the indications are that attitude changes toward mathematics have been invoked that will abet future growth.

The drawing of conclusions though, with respect to the success of the program, must be tempered. Although the program was developed to meet the mathematical need of the underachieving pupil, various factors such as choice of the teacher, teacher enthusiasm, teaching techniques, student selection, class size, content of the material, use of tables and notes, and the uniqueness in which the material was handled make it difficult, if not impossible, to determine any single cause for successful performance.

The program is continuing with the same classes in the 1967-1968 school year. Materials were written during the summer of 1967 based on the experiences encountered with the pupils during the second year of the Exploratory study.

In September, pupils in each class were randomly separated into two groups and re-tested. One group was tested using the SAT Intermediate II tests and the other group was tested using the SAT Advanced tests. This procedure will reveal the amount of regression which takes place over the summer. In addition, using the Advanced tests will allow comparisons to be made which should indicate whether the pupils have advanced far enough mathematically that an advanced test can be used as an indicator of their achievement level.

Pupils will be posttested in the spring using whichever of the two SAT tests are deemed most applicable. Pupils will also be posttested using the SMSG Attitude and Mathematics Inventories.

Information on what mathematics courses these pupils take in their freshman high school year will be collected. It is also planned to record the grades these pupils earn during this freshman year.

In-service seminars for the participating teachers of the Experimental classes are continuing. Emphasis will be placed on content, methods, and means of enabling the pupil to make the transition back to more normal classroom procedures.

APPENDIX 1

PARTICIPATING SCHOOL DISTRICTS, SCHOOLS, AND TEACHERS

Experimental Group

CUPERTINO UNION SCHOOL DISTRICT

Collins School-----Mr. R. Sturtevant
Hyde School--------Mr. J. Fullerton
Miller School------Mrs. A. Bixby

MORELAND SCHOOL DISTRICT

Rogers Junior High School-----Mr. D. Hallstrom

SUNNYVALE ELEMENTARY SCHOOL DISTRICT

Mango Junior High School-----Mrs. S. Webb

SANTA CLARA UNIFIED SCHOOL DISTRICT

Juan Cabrillo School-----Mr. H. Neufeld

FREMONT UNIFIED SCHOOL DISTRICT

Hopkins School-----Mr. E. MacArthur
Walters School-----Mr. R. Masuda

SANTA CRUZ ELEMENTARY SCHOOL DISTRICT

Mission Hill Junior High School-----Mr. D. Herman

LOS GATOS ELEMENTARY SCHOOL DISTRICT

R. J. Fisher School-----Mr. J. Fortier

APPENDIX 2

This appendix contains a table of contents which outlines the material covered by the pupils in the Experimental group. In addition, samples of work sheets and tables have been included.

Work sheet 3-4 is typical of those constructed for use in the program. After a short presentation by the instructor, work sheets, which duplicate the instructor's presentation, are distributed and pupils put quickly to work. Notice that effort has been made to encourage careful reading of the lesson by inserting questions and providing space for answers throughout the exposition of the lesson.

Frequently, because of the nature of the topic, a lesson required a more lengthy presentation. When this was the case, an attempt was made to partially program the lesson. Work sheets 12-1 and 12-1a, which constitute one day's lesson, are examples of this type of presentation.

An example of the review exercises, which are at the end of each unit, is found in work sheet 10-9.

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MULTIPLICATION BY 10, 100, 1000, ...

Look at the multiplication problems below.

a. $1 \cdot 10 = 10$
b. $10 \cdot 10 = 100$
c. $10 \cdot 100 = 1000$
d. $100 \cdot 100 = 10{,}000$
e. $100 \cdot 1{,}000 = 100{,}000$
f. $1{,}000 \cdot 1{,}000 = 1{,}000{,}000$

In problem (c), $10 \cdot 100 = 1{,}000$, how many zeros are there in the numeral 10? ans._____ How many zeros are there in the numeral 100? ans._____ How many zeros are there in the numeral 1,000? ans._____

In problem (e), $100 \cdot 1{,}000 = 100{,}000$, how many zeros are there in the numeral 100? ans._____ How many zeros are the in the numeral 1,000? ans._____ How many zeros are there in the numeral 100,000? ans._____

Do you see any connection between the total number of zeros in the 2 numbers you are multiplying and the number of zeros in the answer? ans._____ Are they the same? ans._____

See if you can finish this statement:

If 1, followed by x number of zeros (x is any whole number) is multiplied by 1, followed by y number of zeros (y is any whole number), then the answer to this multiplication will be 1, followed by _____ + _____ number of zeros.

Fill in the blanks in the problems below.

1. $10 \cdot 10 = \_\_\_\_\_$
2. $100 \cdot 10{,}000 = \_\_\_\_\_$
3. $10{,}000 \cdot 10{,}000 = \_\_\_\_\_$
4. $100{,}000 \cdot 10 = \_\_\_\_\_\_\_\_$
5. $1{,}000 \cdot 10{,}000 = \_\_\_\_\_\_\_\_$
6. $1 \cdot 1 = \_\_\_\_\_$

If, $10 \cdot 100 = 1{,}000$, then $1{,}000 = 10 \cdot 100$, but it is also true
that $1{,}000 = 10 \cdot 10 \cdot 10$, and
that $1{,}000 = 1 \cdot 1{,}000$

Fill in the blanks in the problems below. Problem 8 is done for you.

7. $10 = \_\_\_ \cdot \_\_\_$
8. $100 = \underline{10} \cdot \underline{10} = \underline{1} \cdot \underline{100}$
9. $10{,}000 = \_\_\_ \cdot \_\_\_ = \_\_\_ \cdot \_\_\_ = \_\_\_ \cdot \_\_\_ \cdot \_\_\_ = \_\_\_ \cdot \_\_\_ \cdot \_\_\_ \cdot \_\_\_$
10. Express 100,000 as three different multiplication problems. Consider $1 \cdot 100{,}000$ and $100{,}000 \cdot 1$ as the same problem.
    a.______________________________
    b.______________________________
    c.______________________________

PRODUCT EXPRESSIONS AND FACTORS

Yesterday you learned an easy way to multiply numbers like 10, 100, 1,000, ··· by numbers like 10, 100, 1,000, ··· . For example, you learned that to multiply 1,000 by 10,000, all that was necessary was to total the number of zeros in 1,000, (3), and the number of zeros in 10,000, (4), and that the total, (7), would be the number of zeros that followed the 1 in the answer.

That is: $1,\underbrace{000}_{3\text{ zeros}} \cdot 1\underbrace{0,000}_{4\text{ zeros}} = 1\underbrace{0,000,000}_{7\text{ zeros}}$

There are names, that in order to make talking and writing about multiplication problems easier, you should be familar with.

$$\overbrace{\underbrace{1,000 \cdot 10,000}_{\text{Factors}}}^{\text{Product expression}} = \overbrace{10,000,000}^{\text{Product}}$$

$$\overbrace{\underbrace{10 \cdot 10 \cdot 10}_{\text{Factors}}}^{\text{Product expression}} = \overbrace{1,000}^{\text{Product}}$$

Now you should be ready to make another discovery about numbers like 10, 100, 1,000, ··· . Look at the problems below.

a. 10 = 10
b. 100 = 10 · 10
c. 1,000 = 10 · 10 · 10
d. 10,000 = 10 · 10 · 10 · 10
e. 100,000 = 10 · 10 · 10 · 10 · 10
f. 1,000,000 = 10 · 10 · 10 · 10 · 10 · 10

In the problem 1,000 = 10 · 10 · 10 , how many zeros are there in the product 1,000? ans._____ How many factors are ther in the product expression? ans._____

In the problem 100,000 = 10 · 10 · 10 · 10 · 10 , how many factors are there in the product expression? ans._____ How many zeros are there in the product 100,000? ans._____

Do you see any connection between the number of zeros that follow the 1 in the product and the number of factors in the product expression? ans._____ Are the number of factors in the product expression the same as the number of zeros that follow the 1 in the product? ans._____

(Go on to next page.)

Fill in the blank spaces.

1. If there are 5 factors of only 10's in a product expression, then there will be _____ zeros following the 1 in the product.
2. If there are 13 zeros following a 1 in a product, then there will be _____ factors of 10 in the product expression.

Express the following products as product expressions whose factors are all 10's.

Example: 1,000 = 10 · 10 · 10

3. 10,000 = ______________________________
4. 100 = ______________________________
5. 10,000,000 = ______________________________
6. 100,000 = ______________________________
7. 10,000,000,000,000 = ______________________________
____

Without multiplying, find the product of the following product expressions. Example: 10 · 10 = 100

8. 10 · 10 · 10 · 10 = ______________________________
9. 10 · 10 · 10 = ______________________________
10. 10 · 10 · 10 · 10 · 10 · 10 = ______________________________

Change the product expressions to product expressions of 10's only and then find the product.

Example: 100 · 1,000 =(10 · 10) · (10 · 10 · 10) = 100,000

11. 100 · 10,000 = ______________________________
12. 1,000 · 1,000 = ______________________________
13. 10,000 · 10,000 = ______________________________
14. 100,000 · 100,000 = ______________________________
____

BRAINBUSTER. Without finding the products, how can you show that 10,000 · 100,000 = 1,000 · 1,000,000 ?

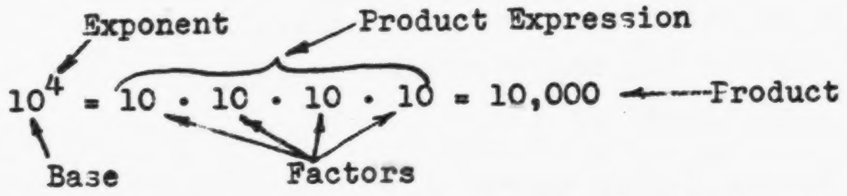
REPEATED FACTORS and EXPONENTS

You have learned that a numeral like 10,000 can be expressed as a product expression that are repeated factors of 10. For example, $10{,}000 = 10 \cdot 10 \cdot 10 \cdot 10$. Is there a short way to write $10 \cdot 10 \cdot 10 \cdot 10$?

Mathematicians have agreed that a short way to write $10 \cdot 10 \cdot 10 \cdot 10$ is 10^4. This new symbol is read, ten to the fourth power . Then $10 \cdot 10$ could be written 10^2, $10 \cdot 10 \cdot 10$ could be written 10^3, for that matter, any expression of repeated factors can be written this short way.

$2 \cdot 2 \cdot 2 = 2^3$	$6 \cdot 6 \cdot 6 = 6^3$
$3 \cdot 3 = 3^2$	$4 \cdot 4 \cdot 4 \cdot 4 = 4^4$
$5 \cdot 5 \cdot 5 \cdot 5 = 5^4$	$9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 = 9^7$

These new names are called exponent forms. When we write $10^4 = 10 \cdot 10 \cdot 10 \cdot 10 = 10{,}000$, the "4" is called the exponent. The "10" is called the base. Study the diagram below.



The exponent tells how many times to use the base as a factor. Factors are to be multiplied.

1. Write each of the following in a short form.

Example: $3 \cdot 3 \cdot 3 \cdot 3 = \underline{3^4}$
Example: $3 + 3 + 3 + 3 = \underline{4 \cdot 3}$

Notice that the first example is multiplication, the second, addition.

a. $2 + 2 + 2 + 2 =$________
b. $10 + 10 + 10 =$________
c. $6 + 6 + 6 + 6 + 6 =$ ________
d. $2 \cdot 2 \cdot 2 \cdot 2 =$______
e. $10 \cdot 10 \cdot 10 =$______
f. $6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 =$______
g. $4 \cdot 4 \cdot 4 =$______
h. $2 \cdot 2 \cdot 2 =$______
i. $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 =$______

(Go on to next page.)

2. Write these to show their meaning.

Example: $2^5 = \underline{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}$

a. $4^3 =$ ____________________

b. $3^4 =$ ____________________

c. $5^2 =$ ____________________

d. $2^3 =$ ____________________

e. $4^5 =$ ____________________

f. $5^4 =$ ____________________

g. $10^5 =$ ____________________

3. Find the value of each of the following expressions.

Example: $4 \cdot 3 = \underline{12}$

Example: $4^3 = 4 \cdot 4 \cdot 4 = \underline{64}$

Notice: $4 \cdot 4 \cdot 4 = 64$

$$\underbrace{\underbrace{4 \cdot 4}_{16} \cdot 4}_{64}$$

a. $3^4 =$ __________

b. $5 \cdot 2 =$ __________

c. $5^2 =$ __________

d. $2^5 =$ __________

e. $2 \cdot 3 =$ __________

f. $2^3 =$ __________

g. $3^2 =$ __________

h. $3 \cdot 2 =$ __________

4. Write each of the following in a shorter manner.

Example: ten to the second power $\underline{10^2}$

Example: ten times two $\underline{10 \cdot 2}$

a. five to the fourth power __________

b. two to the third power __________

c. two times three __________

d. three to the tenth power __________

e. one to the tenth power __________

f. ten to the first power __________

5. Complete the table below.

Power	Product Expression	Product
10^6	$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$	1,000,000
10^5		
10^4		
10^3		
10^2		
10^1		

6. BRAINBUSTER. Do you see any relationship between the exponent for the base "10" and the number of zeros in the product? What is the relationship? ______________________________

____

REGROUPING IN SUBTRACTION

Although all of you know how to subtract, it will be worthwhile for us to review the processes involved in this operation. Consider the problem 68 - 49 = 18.

68 = 6 tens + 8 ones
-49 = 4 tens + 9 ones

Looking ahead, you can see that (8-9) cannot be computed with whole numbers. Therefore it will be necessary to regroup ("borrow") the 6 tens. Regrouping, the problem will look like this.

68 = 6 tens + 8 ones = 5 tens + 18 ones
-49 = 4 tens + 9 ones = 4 tens + 9 ones
19 = 1 ten + 9 ones

Now you can complete the subtraction.

Do the following subtractions as is done in the example above. Do not use the numerals 1, 10, 100, ··· , but write out these numerals in words as is done in the example.

1. 53
-36

2. 75
-37

3. 764
-199

4. 402
-139

5. 710
-287

6. 3,456
-1,567

SUBTRACTION: EXPANDED FORM

The example below shows the subtraction 68 - 49 using expanded form. Study it carefully.

Step 1. 68 and 49 are written in expanded form

$$\begin{array}{r} 68 = 60 + 8 \\ \underline{49 = 40 + 9} \end{array}$$

Step 2. Looking ahead we see that (8-9) cannot be computed with whole numbers. Therefore we regroup 68 as 50 + 18.

$$\begin{array}{r} 68 = 50 + 18 \\ \underline{49 = 40 + 9} \end{array}$$

Step 3. Now we are ready to subtract 9 from 18 and 40 from 50.

$$\begin{array}{r} 68 = 50 + 18 \\ \underline{49 = 40 + 9} \\ 19 = 10 + 9 \end{array}$$

Do the following subtraction problems as is done in the example. Show all the regrouping.

1. $\begin{array}{r} 58 \\ \underline{39} \end{array}$

2. $\begin{array}{r} 73 \\ \underline{56} \end{array}$

3. $\begin{array}{r} 125 \\ \underline{37} \end{array}$

4. $\begin{array}{r} 452 \\ \underline{168} \end{array}$

5. $\begin{array}{r} 503 \\ \underline{247} \end{array}$

6. $\begin{array}{r} 3{,}532 \\ \underline{1{,}654} \end{array}$

SUBTRACTION: SHORT FORM

Study carefully the forms of subtraction below and see if this doesn't explain the "borrowing" you do in subtraction.

Short Form — **Expanded Form**

$$\begin{array}{r} 342 \\ -187 \\ \hline \end{array} \qquad \begin{array}{rcl} 342 &=& 300 + 40 + 2 \\ -187 &=& 100 + 80 + 7 \\ \hline \end{array}$$

$$\begin{array}{r} 3\overset{3}{\cancel{4}}{}^{1}2 \\ -187 \\ \hline \end{array}$$

(2-7) cannot be computed with whole numbers, so we regroup the tens.

$$\begin{array}{rcl} 342 &=& 300 + 30 + 12 \\ -187 &=& 100 + 80 + 7 \\ \hline \end{array}$$

$$\begin{array}{r} \overset{2}{\cancel{3}}\overset{{}^{1}3}{\cancel{4}}{}^{1}2 \\ -187 \\ \hline \end{array}$$

(3-8) cannot be computed with whole numbers, so we regroup the hundreds.

$$\begin{array}{rcl} 342 &=& 200 + 130 + 12 \\ -187 &=& 100 + 80 + 7 \\ \hline \end{array}$$

$$\begin{array}{r} \overset{2}{\cancel{3}}\overset{{}^{1}3}{\cancel{4}}{}^{1}2 \\ -187 \\ \hline 155 \end{array}$$

Now subtract.

$$\begin{array}{rcl} 342 &=& 200 + 130 + 12 \\ -187 &=& 100 + 80 + 7 \\ \hline 155 &=& 100 + 50 + 5 \end{array}$$

Do the following subtraction problems using the short form.

$$1.\ \begin{array}{r} 246 \\ \underline{139} \end{array} \qquad 2.\ \begin{array}{r} 926 \\ \underline{784} \end{array} \qquad 3.\ \begin{array}{r} 964 \\ \underline{777} \end{array} \qquad 4.\ \begin{array}{r} 40 \\ \underline{13} \end{array}$$

$$5.\ \begin{array}{r} 2013 \\ \underline{\ 987} \end{array} \qquad 6.\ \begin{array}{r} 766 \\ \underline{486} \end{array} \qquad 7.\ \begin{array}{r} 949 \\ \underline{892} \end{array} \qquad 8.\ \begin{array}{r} 300 \\ \underline{178} \end{array}$$

$$9.\ \begin{array}{r} 2003 \\ \underline{\ 876} \end{array} \qquad 10.\ \begin{array}{r} 6040 \\ \underline{4793} \end{array}$$

MULTIPLICATION

Now that you know how to multiply, easily, problems like $10 \cdot 5$, $32 \cdot 100$, $\cdots$, and also know that $50 = 5 \cdot 10$, $3200 = 32 \cdot 100$, etc., let's see if you can make another discovery about multiplication. Look at the following multiplication problem then answer the questions that follow.

$$30 \cdot 70 = 2100$$

1. Does $30 = 3 \cdot 10$? ans.____
2. Does $70 = 7 \cdot 10$? ans.____
3. Then does $30 \cdot 70 = (3 \cdot 10) \cdot (7 \cdot 10)$? ans.____
4. Does it make any difference in the answer (product) the way we arrange the factors to be multiplied? That is, does $2 \cdot 3 \cdot 5 = 3 \cdot 5 \cdot 2$? ans.____
5. Then does $30 \cdot 70 = (3 \cdot 10) \cdot (7 \cdot 10) = \underline{3 \cdot 7 \cdot 10 \cdot 10}$? ans.____
6. $3 \cdot 7 =$____
7. $10 \cdot 10 =$________
8. Then does $30 \cdot 70 = (3 \cdot 7) \cdot (10 \cdot 10) = 21 \cdot 100$? ans.____
9. Then the product of $30 \cdot 70 = 21 \cdot 100 =$__________

Do the exercise below and on the next page just as it is done in the examples. Do not leave out any steps.

Examples:

$n = 20 \cdot 30$
$n = (2 \cdot 10) \cdot (3 \cdot 10)$
$n = 2 \cdot 3 \cdot 10 \cdot 10$
$n = 6 \cdot 100$
$n = 600$

$n = 300 \cdot 50$
$n = (3 \cdot 100) \cdot (5 \cdot 10)$
$n = 3 \cdot 5 \cdot 100 \cdot 10$
$n = 15 \cdot 1000$
$n = 15{,}000$

10. $n = \underline{30 \cdot 40}$
$n =$______________
$n =$______________
$n =$______________
$n =$______________

11. $n = \underline{70 \cdot 800}$
$n =$______________
$n =$______________
$n =$______________
$n =$______________

(Go on to next page.)

12. n = $900 \cdot 60$
n = ____________________
n = ____________________
n = ____________________
n = ____________________

13. n = $9 \cdot 400$
n = ____________________
n = ____________________
n = ____________________
n = ____________________

14. n = $600 \cdot 800$
n = ____________________
n = ____________________
n = ____________________
n = ____________________

15. n = $7,000 \cdot 500$
n = ____________________
n = ____________________
n = ____________________
n = ____________________

MULTIPLICATION

Let us see if you have arrived at the quick and easy method of multiplying numbers that end in zeros. Look at the problem below, then answer the questions.

$$3,000 \cdot 700 = 2,100,000$$

1. How many zeros are there in the numeral 3,000 ? ans.______
2. How many zeros are there in the numeral 700 ? ans.______
3. How many zeros altogether ? ans.______
4. How many zeros are there in the product 2,100,000 ? ans.______
5. Do you see any connection? ans.________

Now look at the example again.

$$3,\overbrace{000}^{3\text{ zeros}} \cdot 7\overbrace{00}^{2\text{ zeros}} = \underbrace{2,1}_{(3\cdot7)}\overbrace{00,000}^{(3+2)\text{ zeros}}$$

Study the following examples.
400 · 700 = (4 · 7) followed by 4 zeros = 280000
60 · 30 = (6 · 3) followed by 2 zeros = 1800
8000 · 9000 = (8 · 9) followed by 6 zeros = 72000000

Complete the exercise below using the ideas from the examples above. Use your multiplication tables if you need to.

6. 30 · 500 =____________________
7. 500 · 70 =____________________
8. 300 · 9,000 =____________________
9. 8,000 · 9 =____________________
10. 600 · 900 =____________________
11. 80 · 7,000 =____________________
12. 700 · 700 =____________________
13. 40 · 9,000 =____________________
14. 9,000 · 9,000 =____________________
15. 4,000 · 700 =____________________

LEAST COMMON MULTIPLE-SHORT METHOD

Suppose you were asked to find the least common multiple of 7, 8, and 9. If you were to use the method you have been using the last few days you would find this to be a very long and difficult problem. The least common multiple of 7, 8, and 9 is 504. You would have to list 72 counting numbers that were multiples of 7; 63 counting numbers that were multiples of 8; and 56 counting numbers that were multiples of 9, before you came to the first common multiple. This would take a long time.

Let us see if we can find an easier and quicker method of finding the least common multiple of a set of numbers, for example 8 and 12.

Consider the following:

Does $8 = 2 \cdot 2 \cdot 2$? ans.______

Does $12 = 2 \cdot 2 \cdot 3$? ans.______

Now, let us consider the product expression for 12, that is

$$2 \cdot 2 \cdot 3$$

Is $2 \cdot 2 \cdot 3$ divisible by 12? ans.________

Is $2 \cdot 2 \cdot 3$ divisible by 8? ans.________

In the product expression $2 \cdot 2 \cdot 3$, what other factor is needed so that it is divisible by 8? ans.________

Suppose we take the product expression $2 \cdot 2 \cdot 3$ and place in it another factor of 2. Then the product expression will look like this:

$$2 \cdot 2 \cdot 2 \cdot 3$$

Now, consider the following:

12 will divide $2 \cdot (2 \cdot 2 \cdot 3)$ because the product expression within the parentheses is another name for 12.

8 will divide $(2 \cdot 2 \cdot 2) \cdot 3$ because the product expression within the parentheses is another name for 8. Therefore, the least common multiple of 8 and 12 is $2 \cdot 2 \cdot 2 \cdot 3 = 24$

(go on to next page.)

Exercises. Find the least common multiple of each of the following sets of numbers. Use the short method.

Example. Find the least common multiple of 9 and 15.

$9 = 3 \cdot 3$ Consider $3 \cdot 5$
$15 = 3 \cdot 5$

$3 \cdot 5$ is divisible by 15 because $3 \cdot 5$ is another name for 15.

$3 \cdot 5$ is not divisible by 9. To be divisible by 9, the product expression must have two factors of 3, that is, $3 \cdot 3$.

To make $3 \cdot 5$ divisible by 9, place one more factor of 3 in the product expression, that is, $3 \cdot 3 \cdot 5$.

Now, $3 \cdot 3$ and $3 \cdot 5$ will both divide $3 \cdot 3 \cdot 5$.

Therefore, the least common multiple of 9 and 15 is $3 \cdot 3 \cdot 5 = \underline{45}$

When you write the problem it should look something like this:

$9 = 3 \cdot 3$ L.C.M. $= 3 \cdot 3 \cdot 5 = 45$
$15 = 3 \cdot 5$

1. 12 and 16

2. 14 and 16

3. 10 and 14

4. 16 and 18

5. 12 and 20

6. BRAINBUSTER. 100, 250, and 200

EQUIVALENT FRACTIONS IN LOWER TERMS

In yesterday's exercise you were given a fraction and asked to find an equivalent fraction that had the smallest whole numbers in the numerator and denominator. For example, you were given $\frac{15}{20}$, and as you read up the table you found that $\frac{15}{20} = \frac{12}{16} = \frac{9}{12} = \frac{6}{8} = \frac{3}{4}$, therefore, $\frac{3}{4}$ was your answer as it had the smallest whole numbers in the numerator and denominator of the listed equivalent fractions.

Suppose, though, you were asked to do the same thing with the fraction $\frac{18}{24}$? Your tables do not go that high. Here is a method you can use to reduce a fraction to simplest form, that is, to a form where the fraction has the smallest possible whole numbers in the numerator and denominator.

Consider the fraction $\frac{15}{20}$.

$\frac{15}{20} = \frac{3 \cdot 5}{2 \cdot 2 \cdot 5}$. But $\frac{5}{5}$ is another name for 1.

Then $\frac{15}{20} = \frac{3}{2 \cdot 2} \cdot \frac{5}{5} = \frac{3}{2 \cdot 2} \cdot 1 = \frac{3}{2 \cdot 2} = \frac{3}{4}$

Let's try $\frac{18}{24}$. You fill in the blanks.

The complete factorization of 18 is ______________________________

The complete factorization of 24 is ______________________________

Then $\frac{18}{24} =$ ______________________

But $\frac{2}{2}$ and $\frac{3}{3}$ are but different names for ________.

Then $\frac{18}{24} =$ ________ $\cdot 1 \cdot 1$ but since the product of 1 and any number is that number,

$\frac{18}{24} =$ ____________ $=$ ______________

Exercises. Put the following fractions in simplest form. Use the method shown in the example. Use table 8 to help you.

Example: $\frac{30}{24} = \frac{2 \cdot 3 \cdot 5}{2 \cdot 3 \cdot 2 \cdot 2} = \frac{2}{2} \cdot \frac{3}{3} \cdot \frac{5}{2 \cdot 2} = \frac{5}{4}$

1. $\frac{9}{21}$

(Go on to next page.)

2. $\frac{14}{16}$

3. $\frac{12}{24}$

4. $\frac{15}{45}$

5. $\frac{24}{42}$

6. $\frac{40}{60}$

7. $\frac{75}{100}$

8. $\frac{55}{121}$

9. $\frac{123}{321}$

10. BRAINBUSTER. $\frac{111111}{1111}$

EQUIVALENT FRACTIONS IN HIGHER TERMS

If we can reduce or place a fraction in simplest form by removing the common factors of the numerator and denominator, that is, those factors whose quotient is 1, then by reversing the procedure we can express any fraction as an equivalent fraction of higher terms by multiplying by 1.

Example: $\frac{3}{4} = \frac{3}{4} \cdot \frac{5}{5} = \frac{3 \cdot 5}{4 \cdot 5} = \frac{15}{20}$

Notice we have multiplied the numerator and denominator by the same number, that is, 5, but $\frac{5}{5}$ is but another name for 1, therefore, we have not changed the value of the fraction, only its name.

Exercises.

1. Find the other names for 1, that make the following statements true. Example:

   $\frac{2}{3} \cdot \frac{?}{?} = \frac{10}{15}$ ans. $\frac{2}{3} \cdot \frac{5}{5} = \frac{10}{15}$

   a. $\frac{4}{5} \cdot \underline{\qquad} = \frac{20}{25}$

   b. $\frac{3}{4} \cdot \underline{\qquad} = \frac{24}{32}$

   c. $\frac{4}{3} \cdot \underline{\qquad} = \frac{36}{27}$

   d. $\frac{7}{8} \cdot \underline{\qquad} = \frac{42}{48}$

   e. $\frac{3}{10} \cdot \underline{\qquad} = \frac{300}{1000}$

2. Place the correct number in the numerator or denominator to make the following statements true.

   a. $\frac{2}{3} = \frac{\quad}{15}$

   b. $\frac{3}{4} = \frac{21}{\quad}$

   c. $\frac{7}{5} = \frac{\quad}{25}$

   d. $\frac{1}{2} = \frac{9}{\quad}$

   e. $\frac{1}{1} = \frac{\quad}{24}$

   f. $\frac{5}{8} = \frac{25}{\quad}$

   g. $\frac{7}{9} = \frac{\quad}{63}$

   h. $\frac{11}{12} = \frac{132}{\quad}$

   i. $\frac{7}{10} = \frac{\quad}{100}$

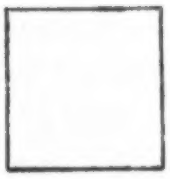
Review exercise

1. The decimal numeral for XXIV is ________.
2. Which is the larger number, 5^3 or 3^5 ? ans.______
3. Using set notation, describe the set of counting numbers that are multiples of 10. ______________________
4. How many different prime factors does 100 have? ans. _____
5. Without dividing, is 10101 divisible by 3? ans._____
6. What is the least common multiple of 2, 4, and 10? ans._____
7. If a number has 10 as a factor, what other factors must the number have? ans.________
8. What is the greatest common factor of 12 and 18? ans._____
9. Put 2,345 in expanded form using exponents.
   2,345 = ______________________________
10. Find all the factors of 36. ans.______________________
11. To change $\frac{4}{5}$ to the equivalent fraction $\frac{28}{35}$, you multiply both numerator and denominator by _____.
12. Reduced to simplest form, $\frac{63}{81}$ = ________
13. If you make a tracing of one figure and place it on top of another figure and if it fits exactly, then we say that the two figures are__________________.
14. In the numeral 234_{five}, the 2 stands for two ______________.
15. The decimal system uses ______(how many) symbols. These symbols are called _________.

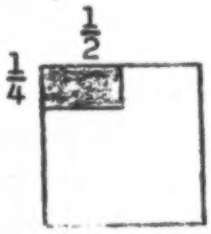
Use the space below to do your work.

PRODUCTS AS MEASURES OF RECTANGULAR REGIONS

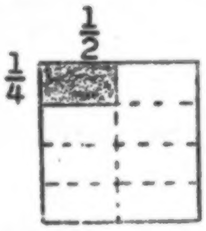
1. Figure A shows a unit square region. Figure B shows the same unit square region. It also shows a shaded rectangular region whose sides are $\frac{1}{4}$ unit and $\frac{1}{2}$ unit in length.



A



B



C

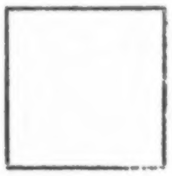
You can separate the unit square region into rectangular regions which are congruent to the $\frac{1}{4}$ by $\frac{1}{2}$ region, as shown in Figure C.

a. How many congruent regions are there? ans.________

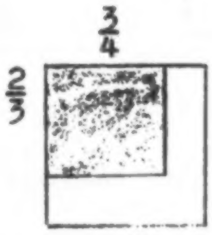
b. How many congruent regions are shaded? ans.________

c. What fraction is represented by Figure C? ans.______

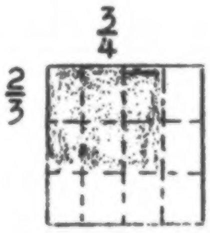
2. Figure D shows a $\frac{2}{3}$ by $\frac{3}{4}$ rectangular region shaded. Figure E shows the unit square region separated into congruent rectangular regions.



A



D



E

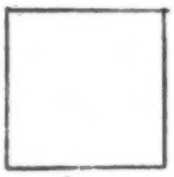
a. In Figure E, how many small rectangular regions are there in the unit square region? ans. _______

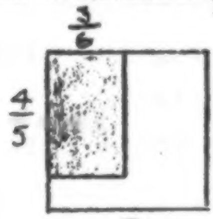
b. How many congruent regions are shaded? ans._____

c. What fraction is represented by Figure E? ans._____

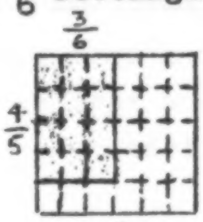
(Go on to next page.)

3. Figures F and G show a shaded $\frac{4}{5}$ by $\frac{3}{6}$ rectangular region.


A



F



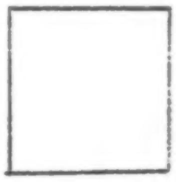
G

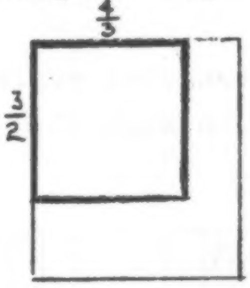
a. In Figure G, how many small congruent rectangular regions are there in the unit square region? ans._____

b. How many of these congruent regions are there? ans._____

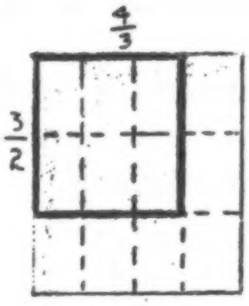
c. What fraction is represented by Figure G? ans._____

4. Figure J shows a shaded rectangular region larger than the unit square region. The unit square is shown in dark lines. Figure J is a $\frac{3}{2}$ by $\frac{4}{3}$ region. Figure K shows the shaded rectangular region and the unit square region separated into congruent rectangular regions.


A



J



K

a. How many small rectangular regions are there in the unit square region? ans.______

b. How many small rectangular regions are there in the $\frac{3}{2}$ by $\frac{4}{3}$ shaded region? ans._____

c. What fraction is represented by Figure K? ans._____

5. Complete this table about the shaded regions in Exercises 1 - 4.

Measure of sides	Fraction represented
$\frac{1}{4}$ by $\frac{1}{2}$	__________
$\frac{2}{3}$ by $\frac{3}{4}$	__________
$\frac{4}{5}$ by $\frac{3}{6}$	__________
$\frac{3}{2}$ by $\frac{4}{3}$	__________

Suppose we wish to change the mixed numeral $5\frac{1}{3}$ to a fraction. The procedure is just the reverse of what we did yesterday. Study the examples.

Example (a)

$5\frac{1}{3} = \frac{(3\cdot 5)+1}{3} = \frac{15+1}{3} = \frac{16}{3}$ check: $\frac{16}{3} = \begin{array}{r} 5 \\ 3\overline{)16} \\ \underline{15} \\ 1 \end{array} = 5\frac{1}{3}$

Example (b)

$6\frac{7}{8} = \frac{(8\cdot 6)+7}{8} = \frac{48+7}{8} = \frac{55}{8}$ check: $\frac{55}{8} = \begin{array}{r} 6 \\ 8\overline{)55} \\ \underline{48} \\ 7 \end{array} = 6\frac{7}{8}$

1. Change the following mixed numerals to fractions and then check your work exactly as in the examples.

   a. $7\frac{5}{8}$

   b. $1\frac{7}{16}$

   c. $7\frac{5}{6}$

   d. $9\frac{6}{7}$

   e. $8\frac{7}{10}$

   f. $13\frac{2}{3}$

   g. $9\frac{3}{4}$

   h. $23\frac{4}{5}$

   i. $12\frac{3}{8}$

   j. $4\frac{15}{16}$

ESTIMATING THE PRODUCT OF TWO MIXED NUMERALS

It is frequently to your advantage to be able to estimate the product of two mixed numerals. The following is one way of making a fairly close estimation.

Before going on though, the symbol > means ________________ and the symbol < means ________________. When using these symbols, the arrow always points toward the __________number.

Now, let us use these two symbols to help us estimate the product of $3\frac{1}{3} \cdot 4\frac{1}{2}$. Answer the following questions by filling in the blanks.

1. What is the first whole number greater than $3\frac{1}{3}$? ans._____
2. What is the first whole number less than $3\frac{1}{3}$? ans._____
3. Is it true that $4 > 3\frac{1}{3} > 3$? ans._____
4. What is the first whole number greater than $4\frac{1}{2}$? ans._____
5. What is the first whole number less than $4\frac{1}{2}$? ans._____
6. Is it true that $5 > 4\frac{1}{2} > 4$? ans._____

Now, let us write the mathematical sentences in questions 3 and 4, one over the other, just as if you were going to multiply vertically, which we are.

$$\begin{array}{c} 5 > 4\frac{1}{2} > 4 \\ 4 > 3\frac{1}{3} > 3 \\ \hline 20 > ? > 12 \end{array}$$

This tells you that the product of $3\frac{1}{3}$ and $4\frac{1}{2}$, which is represented by the question mark (?), will be somewhere between 12 and 20. With a little practice you will be able to do this estimation mentally.

7. Using the method shown above and in problem (a), find two whole numbers between which the product will fall.

a. $6\frac{1}{2} \cdot 9\frac{7}{8}$

$$\begin{array}{c} 10 > 9\frac{7}{8} > 9 \\ 7 > 6\frac{1}{2} > 6 \\ \hline 70 > ? > 54 \end{array}$$

The product will fall between 70 and 54.

(Go on to next page.)

b. $4\frac{1}{3} \cdot 7\frac{4}{5}$

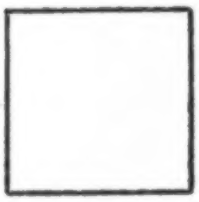
c. $6\frac{7}{8} \cdot 5\frac{1}{2}$

d. $3\frac{3}{4} \cdot 9\frac{1}{3}$

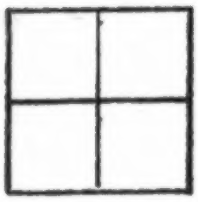
e. $6\frac{1}{2} \cdot 3\frac{1}{3}$

8. Find the products of problems a-e in exercise 7.

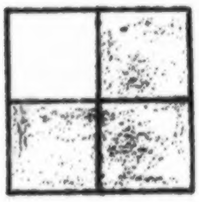
Consider the figures below.



A



B



C

Figure A is a unit square region. Figure B shows the same unit square region divided into 4 smaller congruent rectangular regions. Can each of these smaller regions be thought of as making up $\frac{1}{4}$ of the unit square region? ans._____ Figure C shows the same unit square region with 3 of the smaller regions shaded. What fraction is associated with figure C? ans._____ If each of the smaller regions can be thought of as making up $\frac{1}{4}$ of the unit square region then can the shaded portion in figure C be thought of as three $\frac{1}{4}$s ? ans._____ If it can, then

$$\frac{3}{4} = 3 \cdot \frac{1}{4}$$

1. Express each of the following fractions as a product expression. One of the factors is to be a whole number, the other factor a fraction. Problem (a) and (b) are done for you.

   a. $\frac{1}{3} = 1 \cdot \frac{1}{3}$

   b. $\frac{5}{6} = 5 \cdot \frac{1}{6}$

   c. $\frac{7}{8}$

   d. $\frac{10}{7}$

   e. $\frac{9}{10}$

   f. $\frac{13}{8}$

   g. $\frac{1}{5}$

   h. $\frac{23}{22}$

   i. $\frac{2}{3}$

   j. $\frac{15}{16}$

2. Earlier this year you learned that you could write $5 \div 3$ as $\frac{5}{3}$. The same idea can be used to write $\frac{3}{2} \div \frac{2}{5}$, that is, you can write

$$\frac{\frac{3}{2}}{\frac{2}{5}}$$

(Go on to next page.)

Notice that the bar between $\frac{3}{2}$ and $\frac{2}{5}$ is longer than the other two bars. Write the following as fractions. Problem (a) is done for you.

a. $\frac{3}{4} \div \frac{2}{3} = \dfrac{\frac{3}{4}}{\frac{2}{3}}$

b. $\frac{5}{6} \div \frac{4}{7}$

c. $\frac{9}{8} \div \frac{8}{7}$

c. $\frac{3}{4} \div \frac{6}{7}$

d. $\frac{8}{9} \div \frac{5}{4}$

e. $\frac{13}{7} \div \frac{9}{4}$

3. Write the following as fractions, then write the numerator and denominator of the fraction as a product expression of two factors, one a whole number and one a fraction. Problem (a) is done for you.

a. $\frac{3}{4} \div \frac{5}{7} = \dfrac{\frac{3}{4}}{\frac{5}{7}} = \dfrac{3 \cdot \frac{1}{4}}{5 \cdot \frac{1}{7}}$

b. $\frac{2}{3} \div \frac{3}{4}$

c. $\frac{5}{6} \div \frac{8}{9}$

d. $\frac{3}{8} \div \frac{6}{7}$

What is another name for $\frac{3}{3}$? ans._____ What is another name for $\frac{9}{9}$? ans._____ What is another name for $\frac{235}{235}$? ans._____ Except for zero, what is another name for any number divided by itself? ans._____ Then, what is another name for:

$\frac{\frac{1}{2}}{\frac{1}{2}}$? ans._____ $\frac{\frac{2}{3}}{\frac{2}{3}}$? ans. _____

Using this idea of 1 and the fact that a fraction can be expressed as a product expression of 2 factors, one of which is a whole number and the other a fraction, we can now do division of fractions that have a common denominator. Study the examples carefully.

Example (a)

$$\frac{3}{5} \div \frac{4}{5} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3 \cdot \frac{1}{5}}{4 \cdot \frac{1}{5}} = \frac{3}{4} \cdot \frac{\frac{1}{5}}{\frac{1}{5}} = \frac{3}{4} \cdot 1 = \frac{3}{4}$$

Example (b)

$$\frac{5}{3} \div \frac{2}{3} = \frac{\frac{5}{3}}{\frac{2}{3}} = \frac{5 \cdot \frac{1}{3}}{2 \cdot \frac{1}{3}} = \frac{5}{2} \cdot \frac{\frac{1}{3}}{\frac{1}{3}} = \frac{5}{2} \cdot 1 = \frac{5}{2} \text{ or } 2\frac{1}{2}$$

1. Perform the following divisions. Follow the examples above. Do not omit any steps.

   a. $\frac{3}{4} \div \frac{1}{4}$

   b. $\frac{7}{8} \div \frac{3}{8}$

   c. $\frac{6}{7} \div \frac{2}{7}$

   d. $\frac{3}{11} \div \frac{12}{11}$

   e. $\frac{13}{9} \div \frac{11}{9}$

2. Look at your answers (quotients) and then at the problem again. Do you see a short way to do the division if the denominators are common?

MULTIPLICATION

Suppose we wish to multiply two numbers in decimal form, for example, (.3) · (.25). We know how to multiply these numbers without the decimal places: 3 · 25 = 75

Just as before, we write

$$(.3) \cdot (.25) \text{ or } \begin{array}{r} .25 \\ \underline{.3} \end{array} \quad \text{but}$$

$.3 = \frac{3}{10}$, and $.25 = \frac{25}{100}$, so

$$(.3) \cdot (.25) = \frac{3}{10} \cdot \frac{25}{100} = \frac{75}{1000} = .075$$

Notice that the number of zeros in the denominator of the product, when written as a fraction, is the same as the number of decimal places in the product, when written as a decimal.

Now, answer the following questions about the above problem.

1. How many digits are there to the right of the decimal point in .3 ? ans._____
2. How many digits are there to the right of the decimal point in .25 ? ans._____
3. What is the sum of your answers to (1) and (2)? ans._____
4. How many digits are there to the right of the decimal point in .075 ? ans._____
5. When you compare your answers to (3) and (4), what do you find?____

Consider the multiplication below. Then answer the questions.

$$\begin{array}{r} .735 \\ \underline{.25} \\ 3675 \\ \underline{1470} \\ .18375 \end{array} \qquad \text{thus } (.25) \cdot (.735) = .18375$$

6. How many digits are there to the right of the decimal point in .25 ? ans. _____
7. How many digits are there to the right of the decimal point in .735 ? ans._____
8. What is the sum of your answers to (6) and (7)? ans._____
9. How many digits are there to the right of the decimal point in .18375 ? ans. _____
10. Compare your answers to (8) and (9). What do you find?

(Go on to next page.)

To find the number of decimal places when two numbers are to be multiplied, add the number of decimal places in the two numerals.

Notice that when we multiplied (.25) · (.735) that the first numeral has two decimal places and the second numeral has three decimal places, so there will be five decimal places in the answer. We multiply 25 · 735, and then mark off five decimal places in the answer, counting from right to left.

11. Find the products.
    a. (.8) · (.7)

    b. (.06) · (.9)

    c. (.05) · (.03)

    d. (.4) · (.004)

    e. (.02) · (.007)

    f. (1.3) · (2)

    g. (2.5) · (.3)

    h. (.02) · (1.8)

    i. (1.5) · 61.5)

    j. (2.03) · (.7)

DIVISION

Suppose we wish to divide 1.2 by .3 . How do we divide a decimal by a decimal? You know that $1.2 \div .3$ can be written $\frac{1.2}{.3}$. Now the problem becomes one of how to divide by .3 . Wouldn't it be much easier if the divisor were a counting number, that is, if .3 were just 3? If it were, the division would be quite simple.

When you worked with fractions you learned that you could multiply (or divide) both the numerator and denominator by the same counting number without changing the value of the fraction. Suppose we multiply the numerator (1.2) and the denominator (.3) by 10, then

$$\frac{1.2}{.3} = \frac{1.2}{.3} \cdot \frac{10}{10} = \frac{12.0}{3.0} = \frac{12}{3}\ .$$ Now the division becomes quite simple.

Exercises.

1. Multiply the numerator and denominator of the following numerals by some power of 10 (10, 100, 1,000, etc.) so that the result is a fraction with a counting number as a denominator. Problem (a) is done for you.

   a. $\frac{73.6}{.25} = \frac{73.6}{.25} \cdot \frac{100}{100} = \frac{7360.0}{25.00} = \frac{7360}{25}$

   b. $\frac{.097}{3.26}$

   c. $\frac{685}{8.2}$

   d. $\frac{350}{.007}$

   e. $\frac{.649}{.36}$

Let us now use the idea you learned yesterday to explain how to divide decimals. The example on the left is how you would divide decimals. On the right is what takes place if the division were in the form of a fraction.

(a) $3.36 \div .8$

can be written as

(b) $.8\overline{)3.36}$

Move the decimal point in both the divisor and the dividend the same number of places so that the divisor is a counting number.

$3.36 \div .8$

can be written as

$$\frac{3.36}{.8}$$

(c) $.8\overline{)3.36}$

Do the problem as if it were division of whole numbers, that is, ignore the decimal point.

$$\frac{3.36}{.8} \cdot \frac{10}{10} = \frac{33.60}{8.0}$$

(d)
$$\begin{array}{r} 4\ 2 \\ 8\overline{)33.6} \\ \underline{32} \\ 1\ 6 \\ \underline{1\ 6} \\ 0 \end{array}$$

$$\frac{33.6}{8} = 4\ 2$$

Place the decimal point in the quotient in such a way that the quotient has exactly the same number of decimal points as the revised dividend you got by step (c).

$$\begin{array}{r} 4.2 \\ 8\overline{)33.6} \\ \underline{32} \\ 1\ 6 \\ \underline{1\ 6} \\ 0 \end{array}$$

$$\frac{33.6}{8} = 4.2$$

Of course, with a little practice you will be able to do all of these steps at one time.

(Go on to next page.)

USING YOUR TABLES

This exercise will give you some practice using your Decimal Equivalent Tables.

You have learned that a fraction like $\frac{3}{4}$ has the decimal name .75 . To find this decimal name you did as follows.

$$\frac{3}{4} = \begin{array}{r} .75 \\ 4\overline{)3.00} \\ \underline{2\,8} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

Now $\frac{3}{4}$ can also be thought of as $3 \cdot \frac{1}{4}$. From your tables on page 10, you will find that the decimal name for $\frac{1}{4}$ is .25 . Then it must follow that

$\frac{3}{4} = 3 \cdot \frac{1}{4} = 3 \cdot (.25) = .75$. Let us try another

$\frac{7}{8} = 7 \cdot \frac{1}{8} = 7 \cdot (.125) = .875$ check:

$$\begin{array}{r} .875 \\ 8\overline{)7.000} \\ \underline{6\,4} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

Exercises.

1. Use your table and the above process to find the decimal name for the following fractions, then check to make sure your answers are correct. Problem (a) is done for you.

   a. $\frac{5}{8} = 5 \cdot \frac{1}{8} = 5 \cdot (.125) = .625$ check:

   $$\begin{array}{r} .625 \\ 8\overline{)5.000} \\ \underline{4\,8} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

   b. $\frac{3}{2}$

(Go on to next page.)

b. $\frac{5}{4}$

c. $\frac{3}{8}$

d. $\frac{3}{16}$

e. $\frac{17}{16}$

f. $\frac{13}{20}$

g. $\frac{9}{25}$

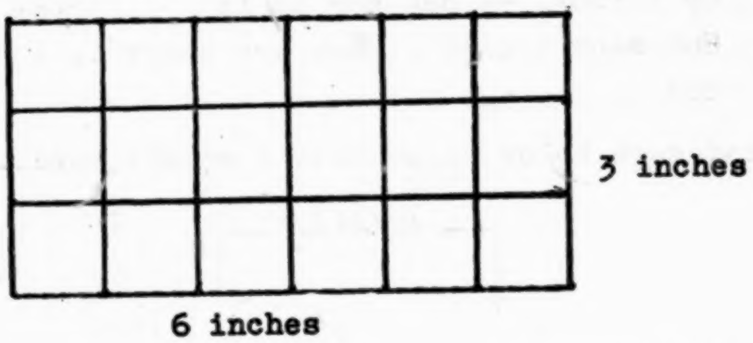
h. **BRAINBUSTER.** $\frac{63}{64}$

AREA OF A RECTANGLE

The chances are that your classroom floor is in the shape of a rectangle and that it is covered with tile. Suppose you were given time enough to count the number of tile on the floor and found there were 800 tiles in all. You are actually finding the area of the floor using tiles as the covering unit.

This is exactly what is done when finding the area of any rectangle only instead of tiles we use a square of some standard measure, that is, an inch, foot, yard, etc.

Suppose we have a rectangle 6 inches long and 3 inches wide and we want to find its area. Because the rectangle is measured in inches, we choose a square, 1 inch on a side, as our covering unit, and see how many of these squares it takes to cover the rectangle.



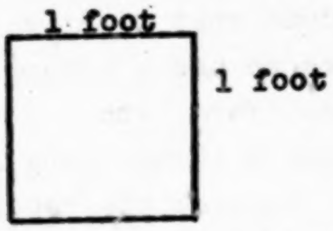
By counting, you will find it takes _____ square inches to cover this rectangle.

1. In the figure above:
   a. How many squares are there in each row (rows go horizontally)? ans._____
   b. How many rows are there? ans._____
   c. Write a mathematical sentence to show how to find the area of the above rectangle without counting.
   ans.________________________________
   d. How many squares are there in each column (columns go vertically)? ans._____
   e. How many columns are there? ans._____
   f. Write a mathematical sentence to show how to find the area of the above rectangle without counting.
   ans.________________________________

(Go on to next page.)

2. If: A = the area of a rectangle
ℓ = the length of the longer side
w = the width (length of the shorter side)
Write a mathematical sentence using A, ℓ, and w to show how to find the area of a rectangle. ans.____________________

3. The figure below represents 1 square foot.

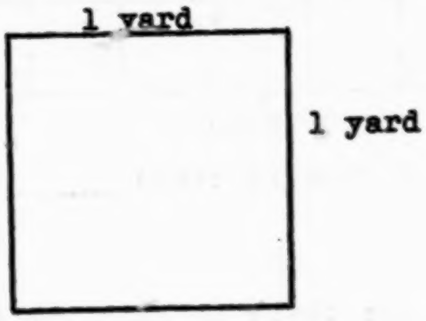


a. How many inches in 1 foot? ans.____

b. Then instead of saying that the above square is 1 foot by 1 foot, we can say it is____inches by____inches.

c. How many square inches are there in 1 square foot? ans.________

4. The figure below represents 1 square yard.



a. How many feet in a yard? ans.____

b. Then instead of saying the above square is 1 yard by 1 yard we can say it is____feet by____feet.

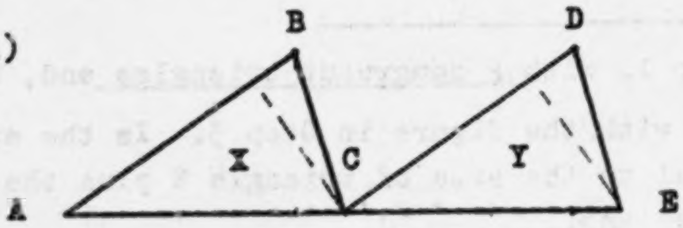
c. How many square feet in 1 square yard? ans.________

d. BRAINBUSTER.
How many square inches in 1 square yard? ans.________

AREA OF A TRIANGLE

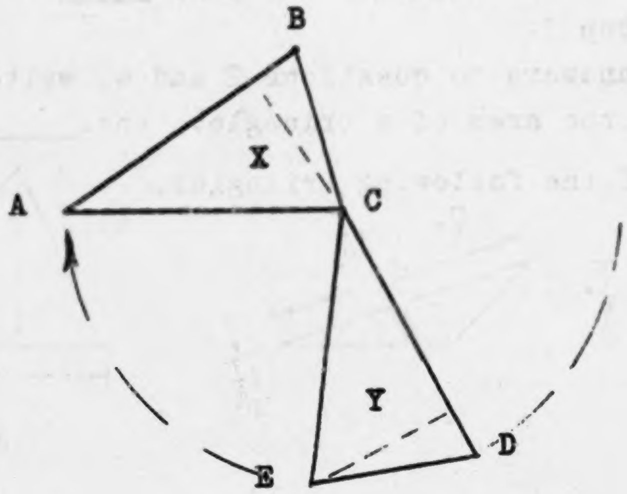
The two triangles below are congruent. (Remember, two figures are congruent if they have exactly the same shape and size.)

Step (1)



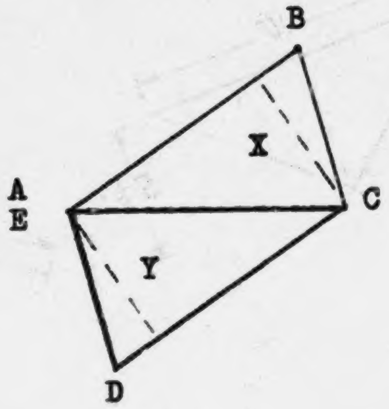
Imagine they are hinged at point C and we start to rotate triangle Y in a clockwise direction, as is shown below.

Step (2)



We continue to rotate triangle Y until point E meets point A, as is shown below.

Step (3)



(Go on to next page.)

Refer to page 19-5 when answering the following questions.

1. In Step 3, what kind of a figure has been formed?
ans.______________________________

2. Write the formula for finding the area of the figure in Step 3. ans.________________________

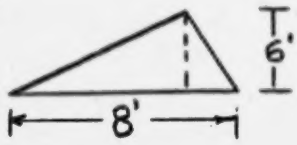
3. We started, in Step 1, with 2 congruent triangles and, by rotating, ended up with the figure in Step 3. Is the area of this figure equal to the area of triangle X plus the area of triangle Y? ans.______

4. Remembering that triangles X and Y are congruent, then the area of 1 of these triangles must be _____ the area of the figure in Step 3.

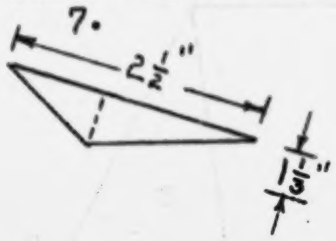
5. Using your answers to questions 2 and 4, write a formula for finding the area of a triangle. ans.__________________

Find the area of the following triangles.

6.



7.



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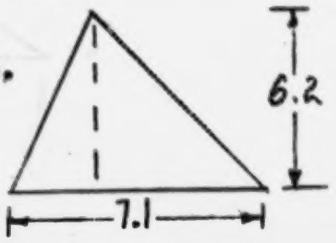

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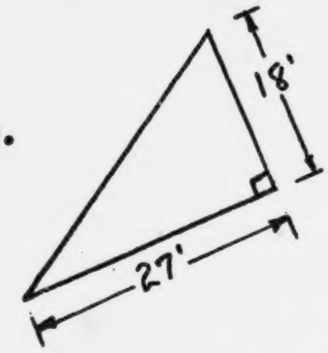
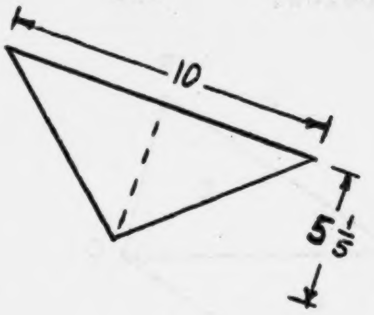
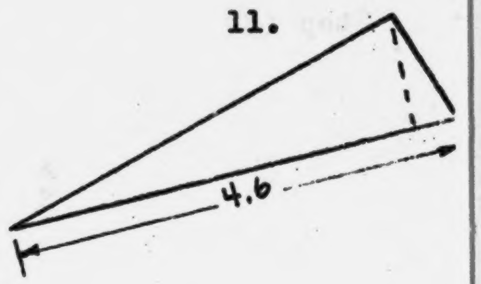


10.



11.



TABLES

TABLE I

ROMAN NUMERALS

Our Numeral	1	5	10	50	100	500	1000
Roman Numeral	I	V	X	L	C	D	M

To write numbers other than those shown above, symbols are repeated, or added.

Example: 3 = III   20 = XX   323 = CCCXXIII

Notice that the symbol for the largest number comes first and is followed, in order, by the symbols for the smaller numbers.

EXCEPTIONS

There are six exceptions to the above rule. When writing a numeral that has a 4 or a 9 in it, subtraction is used.

Below are the six exceptions.

4 = 5 - 1 = IV
9 = 10 - 1 = IX
40 = 50 - 10 = XL
90 = 100 - 10 = XC
400 = 500 - 100 = CD
900 = 1000 - 100 = CM

Example: 444 = 400 + 40 + 4 = CDXLIV
949 = 900 + 40 + 9 = CMXLIX

It is always a good procedure to put a decimal numeral into expanded form before changing it to a Roman numeral.

PLACE VALUE TABLE

GROUP NAME	TRILLION			BILLION			MILLION			THOUSAND			UNITS		
PLACE NAME	hundred	ten	one	hundred	ten	one	hundred	ten	one	hundred	ten	one	hundred	ten	one
DIGITS	1	2	3,	4	5	6,	7	8	9,	8	7	6,	5	4	3

123,
456,
789,
876,
543

one hundred twenty-three trillion,
four hundred fifty-six billion,
seven hundred eighty-nine million,
eight hundred seventy-six thousand,
five hundred forty-three

ADDITION TABLE

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

MULTIPLICATION TABLE

X	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

TABLE of WEIGHTS and MEASURES

LENGTH

12 inches = 1 foot
3 feet = 1 yard
$16\frac{1}{2}$ feet = 1 rod
$5\frac{1}{2}$ yards = 1 rod

$$\left.\begin{array}{r}\text{63,360 inches}\\ \text{5,280 feet}\\ \text{1,760 yards}\\ \text{320 rods}\end{array}\right\} = \text{1 mile}$$

AREA

144 square inches = 1 square foot
9 square feet = 1 square yard
$30\frac{1}{4}$ square yards = 1 square rod
160 square rods = 1 square acre
640 acres = 1 square mile = 1 section

VOLUME

1728 cubic inches = 1 cubic foot
27 cubic feet = 1 cubic yard

LIQUID MEASURE

2 cups = 1 pint
2 pints = 1 quart
4 quarts = 1 gallon
1 gallon = 231 cubic inches

AVOIRDUPOIS WEIGHT

16 ounces = 1 pound
2000 pounds = 1 ton

TIME

60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day
7 days = 1 week
52 weeks = 1 year (approx.)

MISCELLANEOUS

1 acre = 40 yards by 120 yards (approx.)
1 cord = 128 cubic feet
1 hand = 4 inches
1 furlong = 40 rods = 220 yards
1 cubic foot = about $7\frac{1}{2}$ gallons

A SHORT TABLE OF PRIME NUMBERS

2
3
5
7
11
13
17
19
23
29
31
37
41
43
47
53
59
61
67
71
73
79
83
89
97
101
103
107
109
113
127
131
137
139
149
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271
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421
431
433
439
443
449
457
461
463
467
479
487
491
499
503
509
521
523
541
547
557
563
569
571
577
587
593
599
601
607
613
617
619
631
641
643
647
653
659

A prime number is one that has exactly two different factors. For example:

$$5 = 1 \cdot 5$$

$$31 = 1 \cdot 31$$

The following are some divisibility rules for numbers expressed in the decimal system. You will find these very useful in the work to come. After we go over these rules in class, be sure this page is placed in your tables at the back of your binder.

DIVISIBILITY RULES

2: A number is divisible by 2 if the digit in the ones place is a 0, 2, 4, 6, or 8.

Example: 35796 is divisible by 2 because there is a 6 in the ones place.

3: A number is divisible by 3 if the sum of the digits is divisible by 3.

Example: 3,654 is divisible by 3 because 3 + 6 + 5 + 4 = 18, and 18 is divisible by 3.

5: A number is divisible by 5 if there is a 5 or a 0 in the ones place.

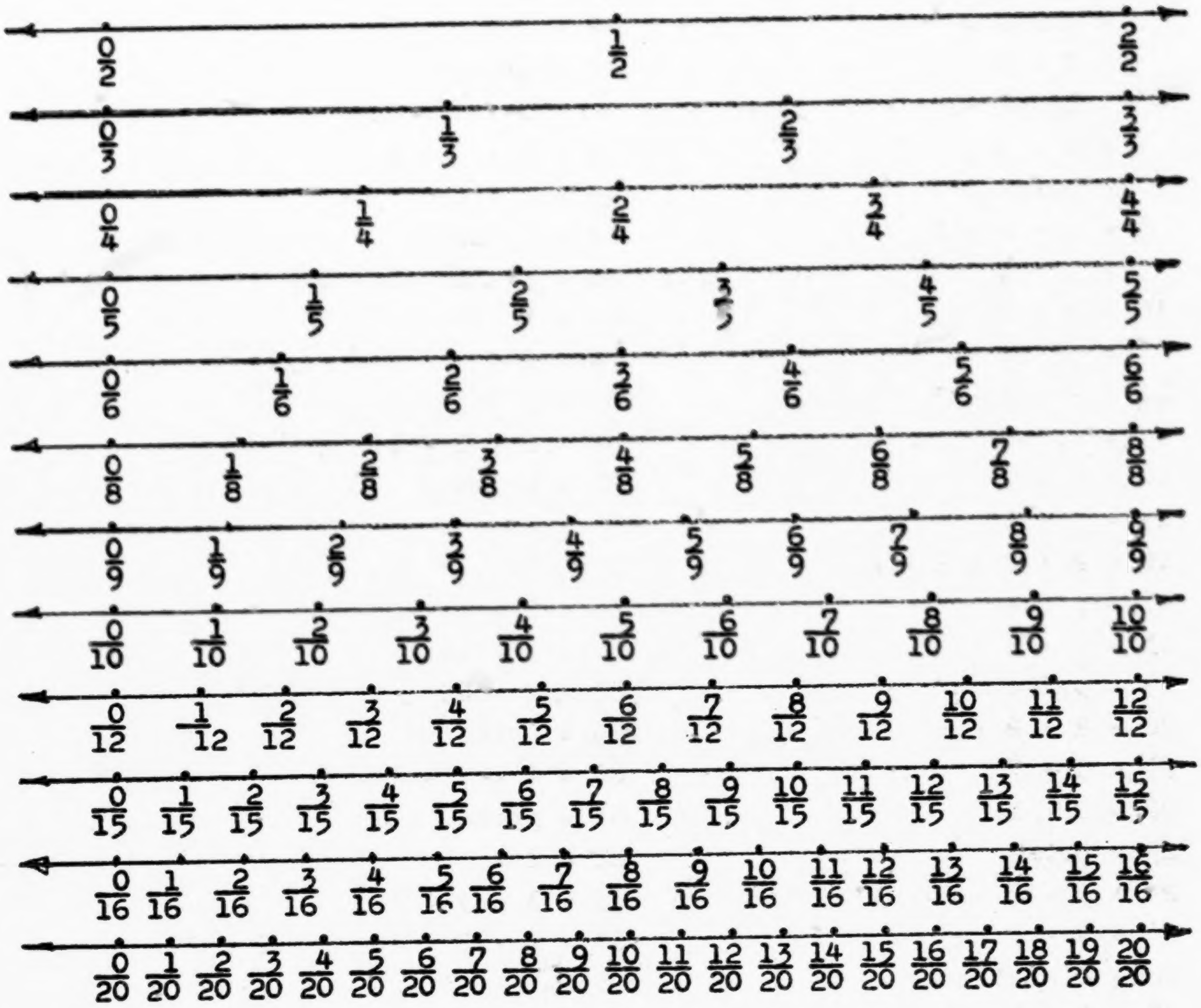
9: A number is divisible by 9 if the sum of the digits is divisible by 9.

Example: 2,106 is divisible by 9 because 2 + 1 + 0 + 6 = 9, and 9 is divisible by 9.

COMPLETE FACTORIZATION TABLE

2 = prime
3 = prime
4 = 2 · 2
5 = prime
6 = 2 · 3
7 = prime
8 = 2 · 2 · 2
9 = 3 · 3
10 = 2 · 5
11 = prime
12 = 2 · 2 · 3
13 = prime
14 = 2 · 7
15 = 3 · 5
16 = 2 · 2 · 2 · 2
17 = prime
18 = 2 · 3 · 3
19 = prime
20 = 2 · 2 · 5
21 = 3 · 7
22 = 2 · 11
23 = prime
24 = 2 · 2 · 2 · 3
25 = 5 · 5
26 = 2 · 13
27 = 3 · 3 · 3
28 = 2 · 2 · 7
29 = prime
30 = 2 · 3 · 5
31 = prime
32 = 2 · 2 · 2 · 2 · 2
33 = 3 · 11
34 = 2 · 17
35 = 5 · 7
36 = 2 · 2 · 3 · 3
37 = prime
38 = 2 · 19
39 = 3 · 13
40 = 2 · 2 · 2 · 5
41 = prime
42 = 2 · 3 · 7
43 = prime
44 = 2 · 2 · 11
45 = 3 · 3 · 5
46 = 2 · 23
47 = prime
48 = 2 · 2 · 2 · 2 · 3
49 = 7 ·7
50 = 2 · 5 · 5
51 = 3 · 17
52 = 2 · 2 · 13
53 = prime
54 = 2 · 3 · 3 · 3
55 = 5 · 11
56 = 2 · 2 · 2 · 7
57 = 3 · 19
58 = 2 · 29
59 = prime
60 = 2 · 2 · 3 · 5
61 = prime
62 = 2 · 31
63 = 3 · 3 · 7
64 = 2 · 2 · 2 · 2 · 2 · 2
65 = 5 · 13
66 = 2 · 3 · 11
67 = prime
68 = 2 · 2 · 17
69 = 3 · 23
70 = 2 · 5 · 7
71 = prime
72 = 2 · 2 · 2 · 3 · 3
73 = prime
74 = 2 · 37
75 = 3 · 5 · 5
76 = 2 · 2 · 19
77 = 7 · 11
78 = 2 · 3 · 13
79 = Prime

MULTIPLE CONGRUENT NUMBER LINES


0/2
1/2
2/2
0/3
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13/20
14/20
15/20
16/20
17/20
18/20
19/20
20/20

TABLE OF DECIMAL EQUIVALENTS

$\frac{1}{2} = .5$

$\frac{1}{3} = .333333$

$\frac{1}{4} = .25$

$\frac{1}{5} = .2$

$\frac{1}{6} = .166666$

$\frac{1}{7} = .142857$

$\frac{1}{8} = .125$

$\frac{1}{9} = .1111111$

$\frac{1}{10} = .1$

$\frac{1}{11} = .090909$

$\frac{1}{12} = .083333$

$\frac{1}{13} = .0769231$

$\frac{1}{14} = .0714286$

$\frac{1}{15} = .0666667$

$\frac{1}{16} = .0625$

$\frac{1}{17} = .0588235$

$\frac{1}{18} = .0555556$

$\frac{1}{19} = .0526316$

$\frac{1}{20} = .05$

$\frac{1}{21} = .047619$

$\frac{1}{22} = .0454545$

$\frac{1}{23} = .0434783$

$\frac{1}{24} = .0416667$

$\frac{1}{25} = .04$

$\frac{1}{26} = .0384615$

$\frac{1}{27} = .037037$

$\frac{1}{28} = .0357143$

$\frac{1}{29} = .0344828$

$\frac{1}{30} = .0333333$

$\frac{1}{31} = .0322581$

$\frac{1}{32} = .03125$

$\frac{1}{33} = .030303$

$\frac{1}{34} = .0294118$

$\frac{1}{35} = .0285714$

$\frac{1}{36} = .0277778$

$\frac{1}{37} = .0270270$

$\frac{1}{38} = .0263158$

$\frac{1}{39} = .0256410$

$\frac{1}{40} = .025$

$\frac{1}{41} = .0243902$

$\frac{1}{42} = .0238095$

$\frac{1}{43} = .0232558$

$\frac{1}{44} = .0227273$

$\frac{1}{45} = .0222222$

$\frac{1}{46} = .0217391$

$\frac{1}{47} = .0212766$

$\frac{1}{48} = .0208333$

$\frac{1}{49} = .0204082$

$\frac{1}{50} = .02$

$\frac{1}{51} = .0196078$

$\frac{1}{52} = .0192308$

$\frac{1}{53} = .0188679$

$\frac{1}{54} = .0185185$

$\frac{1}{55} = .0181818$

$\frac{1}{56} = .0178571$

$\frac{1}{57} = .0175439$

$\frac{1}{58} = .0172414$

$\frac{1}{59} = .0169492$

$\frac{1}{60} = .0166667$

$\frac{1}{61} = .0163934$

$\frac{1}{62} = .0161290$

$\frac{1}{63} = .0158730$

$\frac{1}{64} = .015625$

$\frac{1}{65} = .0153846$

$\frac{1}{66} = .0151515$

$\frac{1}{67} = .0149254$

$\frac{1}{68} = .0147059$

$\frac{1}{69} = .0144928$

$\frac{1}{70} = .0142857$

$\frac{1}{71} = .0140845$

$\frac{1}{72} = .0138889$

$\frac{1}{73} = .0136986$

$\frac{1}{74} = .0135135$

$\frac{1}{75} = .0133333$

$\frac{1}{76} = .0131579$

$\frac{1}{77} = .0129870$

$\frac{1}{78}$ = .0128205

$\frac{1}{79}$ = .0126582

$\frac{1}{80}$ = .0125

$\frac{1}{81}$ = .0123457

$\frac{1}{82}$ = .0121951

$\frac{1}{83}$ = .0120482

$\frac{1}{84}$ = .0119048

$\frac{1}{85}$ = .0117647

$\frac{1}{86}$ = .0116279

$\frac{1}{87}$ = .0114943

$\frac{1}{88}$ = .0113636

$\frac{1}{89}$ = .0112360

$\frac{1}{90}$ = .0111111

$\frac{1}{91}$ = .0109890

$\frac{1}{92}$ = .0108696

$\frac{1}{93}$ = .0107527

$\frac{1}{94}$ = .0106383

$\frac{1}{95}$ = .0105263

$\frac{1}{96}$ = .0104167

$\frac{1}{97}$ = .0103093

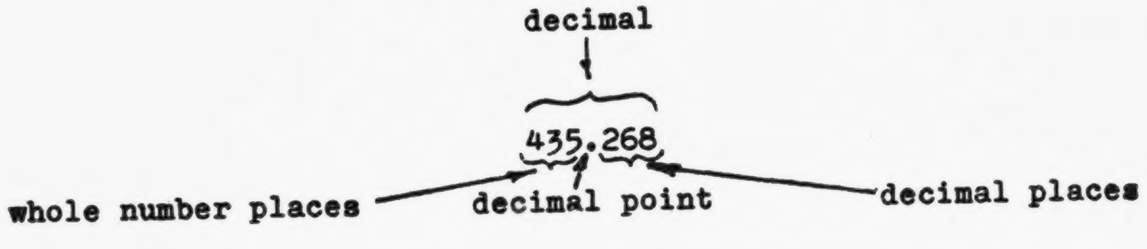
$\frac{1}{98}$ = .0102041

$\frac{1}{99}$ = .0101010

$\frac{1}{100}$ = .01

PLACE VALUE CHART

Hundred thousand	$10\cdot10\cdot10\cdot10\cdot10$ or $10\cdot10{,}000$	10^5	100,000
Ten thousand	$10\cdot10\cdot10\cdot10$ or $10\cdot1{,}000$	10^4	10,000
Thousand	$10\cdot10\cdot10$ or $10\cdot100$	10^3	1,000
Hundred	$10\cdot10$	10^2	100
Ten	$10\cdot1$	10^1	10
One	1	10^0	1
Tenth	$\frac{1}{10}$	$\frac{1}{10^1}$	0.1
Hundredth	$\frac{1}{100}$ or $\frac{1}{10\cdot10}$	$\frac{1}{10^2}$	0.01
Thousandth	$\frac{1}{1{,}000}$ or $\frac{1}{10}\cdot\frac{1}{100}$	$\frac{1}{10^3}$	0.001
Ten-thousandth	$\frac{1}{10{,}000}$ or $\frac{1}{10}\cdot\frac{1}{1{,}000}$	$\frac{1}{10^4}$	0.0001
Hundred-thousandth	$\frac{1}{100{,}000}$ or $\frac{1}{10}\cdot\frac{1}{10{,}000}$	$\frac{1}{10^5}$	0.00001



(Note: be sure this table is placed with your other tables at the back of your binder.)

APPENDIX 3

COMMENTS BY TEACHERS[*]

(Experimental Group)

1. No one is more enthusiastic about the program than I am, among the teaching group. I have no criticism of anything of the content. Where one group may need more drill (examples) concerning any given operation, the teacher can contrive more. Where there are more than necessary, the remainder can be used for games or competition in the group. I can't see cutting or adding on the basis of the needs of one class.

I don't know how my pupils did compared to the rest, but in acceptance and enthusiasm they were, in my opinion, very high. Their formal opinion inventories may prove me wrong but I'd be very much amazed to learn that.

2. This past year of work with this selected group of underachievers has been a very fine experience. The group came to me in September as a collection of disinterested, disorganized, and ignorant students who couldn't care less whether I or anyone else taught them any area of mathematics. Their attention span, at best, was a few minutes.

At first they were somewhat disgruntled about not having a regular textbook, but after several weeks of working on the individual sheets with some degree of success, the textbook problem resolved itself.

After about one month of the program the students were asked if any of them felt that they would rather transfer out of the program and into a regular mathematics class. Without exception, they all wished to remain in the program. As the year progressed, the students' attention span increased and the majority of them showed increasing interest in the worksheets being covered.

About one-third of the way through the year, I was asked to accept a student who was becoming a real problem in another teacher's class. His work was very poor and he was simply making a general nuisance of himself. The interest and response of this boy was amazing. He experienced success immediately and before long excelled over the entire class where he remained the rest of the year.

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[*] These comments appear in a random permutation of the order in which the teachers are listed in Appendix 1.

I believe, the way in which the program was written, the way concepts were developed simply and sequentially, and the fact that students were afforded success as they went along, all contributed toward a very worthwhile experience, both for students and teacher.

Other teachers on the faculty who got glimpses of parts of the materials were very favorably impressed. Their usual question was, "Where can I get my hands on this material?"

Parents were enthused about the program and expressed appreciation for having their children receive this special consideration.

3. First, I must say that I was delighted to be able to use the material and be a part of the SMSG program this past school year. It is much more interesting to me as a teacher than the usual textbook material. I hope, of course, that some of my enthusiasm rubbed off on the students but one can never be sure of matters of that kind. I do know that there are several students in the class who worked hard to do as directed by the material and put all they had into understanding the concepts involved. In every case where the student was motivated to learn I feel confident that they received great value for their efforts.

Unfortunately there were a few who could not be motivated and who were not benefited in the least. Each of the children in this category seemed to have something bothering them beyond just being behind grade level in math.

The ones that were benefited covered a wide range of abilities and had varied backgrounds. In some cases I have heard them talking about math to other students on the school grounds and this in itself seems significant to me.

4. My personal opinion of the program was that it was a great success in my classroom. I am comparing this year's low achieving class with the one I had two years ago. I saw a great difference. My first class did not have the suitable work for their level and this resulted in having more discipline problems and a continuing dislike for math. The material for this year's group was better suited for their level. Many of the lessons could have had a few more problems added to them. This should be done for those students who finished before others.

My main objective was to change the attitude of the students; many of them came to me with a dislike and fear of math. It was felt on the part of both myself and the students that from the experiences we had that this negative attitude was changed to a more positive one.

At times the students felt the lessons were too easy but this was due to past knowledge of the material. I therefore reacted by giving a few more challenging problems.

One thing that I learned from working with the low achiever was that each student needs a great deal of individual attention and he also needs to experience a feeling of success and accomplishment.

I am looking forward to continuing this project next semester.

5. In general I believe the program is proving to be a success. For some reason the make-up of my class was not as I expected. Nine of the students had severe reading problems, so much so that they missed class one day each week for most of the year in order to attend a special remedial reading clinic. Also, ten of the students attended a core class which is for the very poorest students in the seventh grade.

For the above reasons, at least I believe these to be the main reasons, the class did not improve as much as I had hoped. However, some of the students that were of the type that the program was to be for have shown very good improvement.

I think the greatest improvement has been in attitude toward the subject. Almost every student has indicated that they like math more now than they did before.

While the status of the program for next year was in doubt, I had students ask if they would be in the same kind of class next year. I indicated that there was some question as to whether or not the program would continue; when they heard this, they responded with genuine disappointment. Likewise, when they were informed that they would be in the program next year, they were quite happy.

Other teachers in the district have heard of the program and have shown an interest in it. At a district meeting of junior high math teachers I was asked to describe the program. Several teachers were interested in using this as a district program for underachievers in mathematics. This possibility might be of some interest to you and your staff.

6. In evaluating this project I would like to begin by citing the reactions of several groups who were directly or indirectly connected with the program at my school.

The first of these groups are the parents of the students involved. In each and every case of parent-teacher contact (which was about 70 percent of the students' parents) there was, upon explaining the project program, immediate interest and overwhelming cooperation expressed by the parents.

The second of the groups was the other subject-matter teachers of my students, who upon more than one occasion asked, "Why and/or how does Johnny get B's and A's in math and not in my class?" In view of the past background of my students, a most rewarding question.

The last and most important of the groups is the students themselves. In watching the enthusiasm, interest, and achievement shown throughout the year by the students, I found in most cases it was very hard to understand their grade school mathematics careers.

Perhaps more significant was their desire to continue in the program next year. Their relief and pleasure upon hearing this was to be the case was very evident. Undoubtedly part of this can be attributed to their identifying with the teacher, but I am sure a very great deal of this feeling is directly related to the program itself.

Finally, I would like to dwell somewhat on my personal reaction to the program. Having taught several "low" mathematics classes before, both in traditional and SMSG form, I must admit that perhaps I was somewhat skeptical in first approaching this class. However, I soon found that if I was skeptical the students were not, and with their increasing enthusiasm I found myself looking forward to each day of working with them.

It has been a very learning experience, both for the students and myself. Particularly in terms of understanding the spiral curriculum approach, and in discovering that slowing down for a difficult subject area did not help student learning processes. In fact, for overall achievement it was better to maintain an average speed for all units, with student understanding or learning being higher because of the spiral approach.

As for the students, I am convinced that this is an excellent approach in working with this type of underachiever; judging from my student's progress and reactions, I am sure they would agree.

7. I must admit I had a great many doubts as to the degree of success, if any, this new program might enjoy.

Previous to teaching this class, I had felt that whether a student liked or disliked mathematics, or myself, was perfectly irrelevant. I almost knew it made little difference. I believed I could teach mathematics to the students who liked or disliked math or myself with the same degree of success, if any.

I was also a firm believer in very strict discipline. In other words, I believed that the relationship between student and teacher should be more formal than otherwise.

To my surprise the class did extremely well. Unlike other classes of this level they showed much enthusiasm. They also appeared to enjoy the class very much.

From past experiences I knew classes of this level did well to simply get their bodies to class with a pencil. I don't believe I've had any one student forget to bring his supplies (pencil, notebook and sometimes homework) to class more than 3 or 4 times through the whole year. Because a great many students in this class seemed to do well only in math, I found myself at a loss for words when teachers of these students would come to me to ask why this particular student did so poorly in their classes, but so well in mine. All I could tell them, without offending them, was a rather brief explanation of the program.

On numerous occasions I've had parents come to me to express their surprise at the enthusiasm which their children were showing in math this year.

An incident which, to me, exemplifies the general feeling of each student in this class is as follows: A boy in my class was having difficulty in a few of his other classes. When the counselor suggested a change of schedule which necessitated his removal from this math class, he told the counselor, "If I have to change math classes, I don't want my schedule changed."

Because of the great success of the program (in my opinion) I find myself a little confused.

I feel I must sit down and re-evaluate my philosophy as a teacher. I know, because of this year's experience with this special class, I will become a much more effective teacher next year.

8. All in all I felt this was a most successful year for almost all the children in this special class. It was somewhat of a struggle to get some of them to use their addition and multiplication tables. Once they got used to them they really started making progress because they started getting more problems correct and most began then to take a little pride in their work. I found that they needed constant bolstering as to the effect that they were covering seventh grade work and that they were not stupid. I found that the better the self-concept the better they did in math.

All except two of the childrens' parents were represented at the open house at the beginning of the year. They were very pleased with the idea of the program and several felt this type of class should have been available at an earlier age.

I met with all the parents individually in May and we discussed their child's progress in the program. They were all most flattering to the program. They all felt their child either liked math or did not complain about it. They all wanted their children to participate in the program next year. One father even volunteered to write a letter to the people in charge if it would do any good.

The boy who was the most negative at the beginning of the year turned out to be one of the most positive students. His parents were certainly pleased and had noticed his improvement at home.

The counselor talked to almost all the students in the class at one time or another during the year and found a surprising number liked math class best of all. They didn't think it was any noisier than any other class. (This is not my opinion.)

I would say every child got so that they would volunteer information. Of course some were much more enthusiastic than others. It was a wonderful experience to teach this class and I feel I learned a great deal myself.

9. In trying to evaluate working with this SMSG program during this year I keep coming up with one idea. It was exciting. The students enjoyed the program and I enjoyed working with them.

Initially, I was apprehensive. I am not a mathematics teacher, having taught only one year of mathematics and not really doing too well at that. I thought the program sounded interesting and would be a challenge. It was both.

I found the students nice but not very eager, as far as mathematics was concerned. They were pleased not to have mathematics books but not sure about the whole approach. They became interested in the program quickly and never reverted to the way they felt in September.

The parents seemed grateful that their children were chosen for special help. None of the 18 parents that I met objected to the program. One parent considered not allowing his child to stay in the program but when it was explained to him he was most enthusiastic. Another father told me this was the first time he felt his daughter had a chance to be helped and that he wished he had had something like this when he was in school. As I saw the parents through the year they were quite pleased and encouraged about the progress of their children.

The students thrived on their success in the program and not feeling stupid. They were cooperative, lively and worked well. This carried over into other classes in some cases. One girl was moved into a high English class and high social studies class during the year. She was my top student and did very well in all her classes. There were a few students who did well in mathematics but did poorly in their other classes. Success in mathematics was a point of pride with these students. They were successful in something and it was important to them. One boy has decided to teach mathematics when he grows up. He had gotten nothing but D's before this year. He may not have the ability to get through college--but now he thinks he might try.

The other mathematics teachers at my school can probably judge the program more objectively than I can. One of them feels that the program is so valuable that he would like to use it for his low seventh grade classes next year. He sees success in the way the students react to mathematics as well as their achievement.

As the students became more interested in the work, had some success and relaxed, discipline was no problem. Some of the students are very immature, but all of them responded to classroom control very well. I had no real behavior problems though some of the students had trouble in other classes and on the school grounds. I treated them as though they were trustworthy and I expected them to behave well. At first this was not always the case, but by the end of the year they behaved as I expected most of the time.

Beside being a very good year as far as the students were concerned, I have enjoyed this year for personal reasons. I am more aware of the scope of mathematics, the pleasures it can provide and the stimulation of teaching a class in SMSG. I appreciate being selected to participate in this program and I am looking forward to next year.

10. Materials: Generally well laid out. Appropriate simple terminology. Average instructor should be able to make them useful to youngsters with whom we are working. Suggest that more brainbuster type examples be provided at end of each unit and that each daily assignment include at least one brainbuster type problem.

Student Reaction: Youngsters felt good all through the year about being part of this program. They felt it was prestigious to be studying "new math" just as other youngsters in high level classes were doing. Faster youngsters registered impatience, sometimes overtly, when other youngsters required a fuller explanation of a concept than was stated in the materials. Most students wanted to supplement work with mathematics games and voluntarily provided their own from books they had at home or found in libraries. By the end of the year one could generally say that many of the "dull" had seen a little more to math than they ever had before. But, one would also have observed that the "faster" students had become bored almost to the point of impatience. However, planning ahead by the instructor to avert this latter phenomenon could well be done.

Faculty Reaction: Teachers were generally supportive in philosophy, but few took the trouble to find out what was really going on. Those who did so showed guarded interest but did follow up with inquiries on activities and progress from time to time. Instructors in the math department were actively supportive and made frequent inquiries and expressed hope that similar materials would be made available to them in the near future.

Administrative Reaction: The principal was very supportive, philosophically and actively. He asked for a short report at a faculty meeting and followed it up with references to what we are trying to do at other faculty gatherings. He also visited the class while in session and made a point of discussing the program from time to time.

In conclusion, the outstanding weakness of the program is the inadequate treatment of the material by the various instructors, such as myself, who "forgot" the main thesis of one-lesson-one-day and "dragged" the students through concepts just as if they were conducting a conventional math class. By the last two quarters that practice had dropped entirely.

The outstanding strength of the program is the ready accessibility of functional easy-to-use materials.

Thank you for the opportunity of working with this experimental program. It has changed my deliveries and techniques in other classes as well and provided me with tools for the best year I have enjoyed in education yet.

APPENDIX 4

SCALE DESCRIPTIONS -- SMSG MATHEMATICS INVENTORY, FORM SC

SM1 Whole Number Structure 1 (6 items; 4 minutes) This scale is designed to measure knowledge of basic properties such as commutativity, associativity, distributivity, and inverse elements with respect to addition and multiplication of whole numbers.

EXAMPLE: $3 \times 26 = (3 \times \square) + (3 \times 6)$

(A) 2 (B) 6 (C) 20 (D) 26 (E) None of these

SM2 Open Sentences-Operations 1 (6 items; 2 minutes) This scale is intended to measure ability to identify the operation which must be performed in order to find the missing number.

EXAMPLE: $23 \times 51 = \square$

(A) add
(B) subtract
(C) multiply
(D) divide

SM3 Rationals-Non-Computation (11 items) This scale is intended to measure ability to convert rational numbers from one form to another and to make a routine application of knowledge about the rational numbers in solving a problem.

EXAMPLE: $\frac{72}{5}$ equals:

(A) $\frac{7}{5} + \frac{2}{5}$
(B) $\frac{1}{72} \times 5$
(C) $14\frac{2}{5}$
(D) $71\frac{1}{5}$
(E) $7 + \frac{2}{5}$

SM4 Whole Numbers 1 (9 items) This scale is intended to measure understanding of notation and terminology of whole numbers and ability to perform specified operations.

EXAMPLE: The sum of the three numbers 1,125 , 718 , and 1,090 is:

(A) 1,952
(B) 2,823
(C) 2,933
(D) 3,933
(E) 9,395

SM5 Geometry 1 (4 items) This scale is designed to measure understanding of geometric relationships.

EXAMPLE: A straight line and a triangle CANNOT intersect in

(A) exactly 0 points.
(B) exactly 1 points.
(C) exactly 2 points.
(D) exactly 3 points.
(E) an unlimited number of points.

SCALE DESCRIPTIONS -- SMSG OPINION INVENTORY, FORM SC

AT1 Mathematics vs. Non-Mathematics (8 items) This scale is designed to measure how well a student likes mathematics and considers it important in relation to other school subjects.

I like story books _________ than mathematics books.

(A) a lot more
(B) a little more
(C) a little less
(D) a lot less

AT2 Mathematics Fun vs. Dull (4 items) This scale is designed to measure the pleasure or boredom a student experiences with regard to mathematics both in an absolute sense and comparatively with other subjects.

Mathematics is fun.

(A) strongly agree
(B) agree
(C) don't know
(D) disagree
(E) strongly disagree

AT3 Pro-Mathematics Composite (11 items) The scale is designed to measure general attitude toward mathematics. It is an overall scale including items drawn from scales AT1, AT2, and AT4 and other items not used in these scales.

I can get along perfectly well in everyday life without mathematics.

(A) strongly agree
(B) agree
(C) don't know
(D) disagree
(E) strongly disagree

AT4 Mathematics Easy vs. Hard (9 items) This scale is designed to measure the ease or difficulty which a student associates with mathematics performance.

No matter how hard I try, I cannot understand mathematics.

(A) strongly agree
(B) agree
(C) don't know
(D) disagree
(E) strongly disagree

AT5 Ideal Mathematics Self-Concept (8 items) This scale is designed to measure how a child wishes he were in his relationship to mathematics.

I wish it were easier for me to talk in front of my mathematics class.

(A) strongly agree
(B) agree
(C) mildly agree
(D) mildly disagree
(E) disagree
(F) strongly disagree

AT6 Facilitating Anxiety 1 (9 items) This scale is designed to measure the degree to which mathematics achievement performance is facilitated by stressful conditions (e.g., examinations).

I keep my mathematics grades up mainly by doing well on the big tests rather than on homework and quizzes.

(A) always
(B) usually
(C) sometimes
(D) hardly ever
(E) never

AT7 Debilitating Anxiety 1 (10 items) This scale is designed to measure the degree to which mathematics achievement performance is harmed by stressful conditions (e.g., examinations).

When I have been doing poorly in mathematics, my fear of a bad grade keeps me from doing my best.

(A) never
(B) hardly ever
(C) sometimes
(D) usually
(E) always

AT8 *Actual Mathematics Self-Concept* (8 items) This scale is designed to measure how a child sees himself in relation to mathematics.

I find it hard to talk in front of my mathematics class.

(A) strongly agree
(B) agree
(C) mildly agree
(D) mildly disagree
(E) disagree
(F) strongly disagree

APPENDIX 5

INITIAL MEASURES BY CLASS

CLASS NO.*	1		2		3		4		5	
NO. STUDENTS	21		25		26		26		25	
	$\overline{X}$	S.D.	$\overline{X}$	S.D.	$\overline{X}$	S.D.	$\overline{X}$	S.D.	$\overline{X}$	S.D.
SEX**	1.71	.46	1.32	0.48	1.42	0.50	1.62	0.50	1.76	0.44
I.Q.	101.00	9.20	96.12	13.14	100.23	14.34	102.56	6.64	98.32	9.24
COMP.***	44.67	6.67	41.48	8.70	41.73	6.92	42.04	9.93	39.32	6.41
APPL.***	47.90	7.71	47.24	9.44	51.35	13.39	52.65	13.33	50.60	9.92
SMSG 1	2.05	1.28	1.84	1.28	1.27	1.34	1.96	1.37	2.28	1.49
SMSG 2	3.24	1.70	3.04	1.46	3.31	1.85	3.27	1.89	3.16	1.80
SMSG 3	1.14	1.06	1.44	1.23	1.58	1.17	1.50	1.07	1.52	1.19
SMSG 4	3.00	1.41	3.04	1.40	2.73	1.66	3.15	1.49	3.24	1.48
SMSG 5	0.95	0.92	0.56	0.71	0.50	0.95	0.85	0.61	0.68	0.90
ATT. 1	19.90	3.56	20.52	4.04	18.27	4.27	20.38	4.67	20.68	4.48
ATT. 2	12.90	3.27	14.08	3.98	11.50	4.37	12.23	4.38	12.52	4.45
ATT. 3	32.86	3.58	34.52	5.85	31.77	6.42	32.46	5.62	33.12	5.78
ATT. 4	23.10	2.93	26.24	5.82	20.81	5.32	23.19	6.71	23.96	7.13
ATT. 5	35.67	4.27	34.20	6.46	37.31	6.42	37.19	5.35	35.60	5.47
ATT. 6	22.86	4.49	26.58	6.50	23.88	5.64	23.54	5.67	25.40	5.46
ATT. 7	31.38	5.46	28.13	7.70	31.00	5.81	32.92	8.51	30.28	6.81
ATT. 8	28.05	5.41	28.35	6.15	23.77	6.01	24.46	7.44	27.96	7.00

* Classes 1 to 10 are the experimental classes, 11-15 the controls.

** Boy = 1 Girl = 2

*** These scores have been multiplied by 10 for programming purposes.

CLASS NO.*	6		7		8		9		10	
NO. STUDENTS	26		19		21		20		26	
	$\overline{X}$	S.D.	$\overline{X}$	S.D.	$\overline{X}$	S.D.	$\overline{X}$	S.D.	$\overline{X}$	S.D.
SEX**	1.31	0.47	1.21	0.42	1.62	0.50	1.50	0.51	1.38	0.50
I.Q.	107.00	9.65	104.39	6.63	97.85	11.86	97.20	9.03	103.63	10.12
COMP.***	54.88	10.39	52.26	10.34	46.29	8.87	46.85	11.02	50.46	8.35
APPL.***	76.62	12.13	59.05	16.41	50.29	11.72	50.55	8.93	56.50	10.06
SMSG 1	3.42	1.10	2.68	1.73	1.00	1.64	1.60	1.64	2.50	1.63
SMSG 2	4.46	1.70	3.05	1.99	2.19	1.29	3.50	1.79	4.00	1.92
SMSG 3	2.92	1.60	1.89	1.45	1.24	0.83	1.80	1.15	1.46	1.27
SMSG 4	5.46	1.63	3.95	1.78	2.67	1.53	3.80	1.61	4.42	1.45
SMSG 5	1.12	0.95	0.95	1.03	0.57	0.68	0.60	0.68	0.58	0.70
ATT. 1	21.73	5.29	22.11	3.46	20.90	4.37	20.30	4.01	19.58	4.64
ATT. 2	14.46	4.25	15.58	2.91	13.00	3.95	14.00	4.59	12.23	4.45
ATT. 3	35.65	5.26	36.89	4.18	32.10	4.69	34.00	4.99	32.85	5.15
ATT. 4	27.38	6.86	27.89	4.52	25.57	5.15	25.70	5.19	23.92	5.91
ATT. 5	33.92	4.85	32.26	4.68	35.14	5.64	36.40	5.89	36.65	5.74
ATT. 6	25.96	5.41	25.95	4.05	24.29	6.60	25.05	6.06	24.31	3.83
ATT. 7	27.35	5.85	29.68	4.14	27.38	4.94	28.25	9.75	30.88	5.71
ATT. 8	30.77	4.84	31.00	5.49	29.95	4.27	27.32	6.82	28.23	5.54

* Classes 1 to 10 are the experimental classes, 11-15 the controls.

** Boy = 1 Girl = 2

*** These scores have been multiplied by 10 for programming purposes.

CLASS NO.*	11		12		13		14		15	
NO. STUDENTS	21		28		19		23		25	
	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.
SEX**	1.48	0.51	1.54	0.51	1.42	0.51	1.52	0.51	1.52	0.51
I.Q.	102.41	12.26	109.57	11.46	95.44	11.44	105.44	8.82	95.23	7.26
COMP.***	43.10	7.94	49.18	9.00	36.84	7.22	54.83	8.57	47.04	8.34
APPL.***	53.95	12.05	58.07	13.12	58.05	12.05	68.74	14.94	52.04	8.39
SMSG 1	2.14	1.31	2.38	1.34	1.42	1.47	3.43	1.41	2.00	1.53
SMSG 2	3.19	1.91	3.39	2.08	2.21	1.58	4.26	1.79	3.76	2.05
SMSG 3	1.29	1.23	2.14	1.74	1.89	1.24	2.52	1.44	1.40	1.12
SMSG 4	3.57	1.86	3.68	1.85	3.21	1.62	4.91	2.11	3.68	1.65
SMSG 5	0.43	0.75	0.82	0.94	0.68	0.95	0.83	0.78	0.72	0.74
ATT. 1	18.10	6.33	19.43	4.65	20.68	4.26	21.48	5.16	19.96	7.56
ATT. 2	11.52	5.13	13.07	3.55	12.37	3.50	15.17	4.76	13.08	4.40
ATT. 3	29.90	8.34	31.93	5.13	30.68	4.74	35.39	5.56	30.60	8.38
ATT. 4	21.19	8.16	24.82	4.60	24.21	4.57	26.70	6.61	24.24	8.40
ATT. 5	33.62	9.54	34.86	3.70	35.95	3.72	35.04	6.67	33.16	8.76
ATT. 6	19.90	5.63	25.50	4.84	24.68	5.75	27.91	5.17	24.80	7.51
ATT. 7	29.05	10.59	28.43	5.97	31.84	6.14	27.57	5.09	28.72	8.38
ATT. 8	26.45	10.68	29.89	5.68	27.26	4.93	30.26	6.84	28.04	8.18

* Classes 1 to 10 are the experimental classes, 11-15 the controls.

** Boy = 1 Girl = 2

*** These scores have been multiplied by 10 for programming purposes.

APPENDIX 6

INITIAL MEASURES BY GROUPS

	EXPERIMENTAL(N=235)		CONTROL(N=116)	
	$\overline{X}$	S.D.	$\overline{X}$	S.D.
SEX*	1.49	0.50	1.50	0.50
I.Q.	100.83	10.67	101.89	11.66
COMP.**	45.86	10.05	46.72	9.99
APPL.**	54.05	13.49	56.50	13.92
SMSG 1	2.08	1.58	2.30	1.53
SMSG 2	3.35	1.81	3.41	1.99
SMSG 3	1.66	1.30	1.86	1.44
SMSG 4	3.56	1.74	3.83	1.88
SMSG 5	0.73	0.83	0.71	0.83
ATT. 1	20.39	4.40	19.91	5.76
ATT. 2	13.17	4.21	13.09	4.39
ATT. 3	33.56	5.40	31.76	6.79
ATT. 4	24.68	6.03	24.31	6.78
ATT. 5	35.51	5.64	34.48	6.83
ATT. 6	24.77	5.46	24.68	6.29
ATT. 7	29.79	6.61	29.00	7.43
ATT. 8	27.85	6.34	28.54	7.46
DAYS ABS.	8.38	7.30	6.86	6.64
UNITS COVERED	12.79	2.85	15.74	3.06

* Boy = 1 Girl = 2

** These scores have been multiplied by 10 for programming purposes.

APPENDIX 7

ANALYSIS OF VARIANCE -- INITIAL SCORES

```
        UNIVARIATE ANOVA ON -- SEX

************************************************************
SOURCE OF
VARIATION               SS           DF       MS          F
************************************************************

BETWEEN                .00            1      .00        .00
WITHIN               79.00          314      .25

     TOTAL           79.00          315
************************************************************
```

```
        UNIVARIATE ANOVA ON -- STUDENT IQ

************************************************************
SOURCE OF
VARIATION               SS           DF       MS          F
************************************************************

BETWEEN              74.85            1    74.85        .62
WITHIN            37873.18          314   120.62

     TOTAL        37948.03          315
************************************************************
```

```
        UNIVARIATE ANOVA ON -- PRE COMP

************************************************************
SOURCE OF
VARIATION               SS           DF       MS          F
************************************************************

BETWEEN              49.00            1    49.00        .48
WITHIN            34566.31          341   101.37

     TOTAL        34615.30          342
************************************************************
```

```
      UNIVARIATE ANOVA ON -- PRE APPL

*************************************************************
SOURCE OF
VARIATION                  SS          DF        MS         F
*************************************************************

BETWEEN                376.77           1    376.77      2.02
WITHIN               63677.87         341    186.74

    TOTAL            64054.65         342
*************************************************************

      UNIVARIATE ANOVA ON -- PRE-SMSG 1

*************************************************************
SOURCE OF
VARIATION                  SS          DF        MS         F
*************************************************************

BETWEEN                  2.47           1      2.47      1.01
WITHIN                 833.72         341      2.44

    TOTAL              836.19         342
*************************************************************

      UNIVARIATE ANOVA ON -- PRE-SMSG 2

*************************************************************
SOURCE OF
VARIATION                  SS          DF        MS         F
*************************************************************

BETWEEN                   .44           1       .44       .12
WITHIN                1208.29         341      3.54

    TOTAL             1208.73         342
*************************************************************
```

```
          UNIVARIATE ANOVA ON -- PRE-SMSG 3

************************************************************
SOURCE OF
VARIATION              SS           DF        MS          F
************************************************************

BETWEEN              3.19            1       3.19       1.75
WITHIN             621.25          341       1.82

     TOTAL         624.44          342
************************************************************
```

```
          UNIVARIATE ANOVA ON -- PRE-SMSG 4

************************************************************
SOURCE OF
VARIATION              SS           DF        MS          F
************************************************************

BETWEEN              3.90            1       3.90       1.22
WITHIN            1092.55          341       3.20

     TOTAL        1096.44          342
************************************************************
```

```
          UNIVARIATE ANOVA ON -- PRE-SMSG 5

************************************************************
SOURCE OF
VARIATION              SS           DF        MS          F
************************************************************

BETWEEN               .02            1        .02        .02
WITHIN             237.77          341        .70

     TOTAL         237.78          342
************************************************************
```

```
          UNIVARIATE ANOVA ON -- PRE-ATTI 1

************************************************************
SOURCE OF
VARIATION              SS           DF        MS          F
************************************************************

BETWEEN             17.30            1      17.30        .72
WITHIN            8156.70          341      23.92

     TOTAL        8174.00          342
************************************************************
```

```
         UNIVARIATE ANOVA ON -- PRE-ATTI 2

*************************************************************
SOURCE OF
VARIATION                SS          DF        MS          F
*************************************************************

BETWEEN                  .33          1        .33        .02
WITHIN               6196.73        341      18.17

     TOTAL           6197.06        342
*************************************************************


         UNIVARIATE ANOVA ON -- PRE-ATTI 3

*************************************************************
SOURCE OF
VARIATION                SS          DF        MS          F
*************************************************************

BETWEEN               247.89          1     247.89       7.03
WITHIN              12018.14        341      35.24

     TOTAL          12266.03        342
*************************************************************


         UNIVARIATE ANOVA ON -- PRE-ATTI 4

*************************************************************
SOURCE OF
VARIATION                SS          DF        MS          F
*************************************************************

BETWEEN                11.19          1      11.19        .29
WITHIN              13185.67        341      38.67

     TOTAL          13196.86        342
*************************************************************


         UNIVARIATE ANOVA ON -- PRE-ATTI 5

*************************************************************
SOURCE OF
VARIATION                SS          DF        MS          F
*************************************************************

BETWEEN                93.97          1      93.97       2.63
WITHIN              12196.03        341      35.77

     TOTAL          12290.00        342
*************************************************************
```

```
          UNIVARIATE ANOVA ON -- PRE-ATTI 6

*********************************************************
SOURCE OF
VARIATION                  SS           DF       MS        F
*********************************************************

BETWEEN                  4.10           1      4.10      .14
WITHIN               10260.90         341     30.09

     TOTAL           10265.00         342
*********************************************************
```

```
          UNIVARIATE ANOVA ON -- PRE-ATTI 7

*********************************************************
SOURCE OF
VARIATION                  SS           DF       MS        F
*********************************************************

BETWEEN                 65.76           1     65.76     1.46
WITHIN               15340.64         341     44.99

     TOTAL           15406.40         342
*********************************************************
```

```
          UNIVARIATE ANOVA ON -- PRE-ATTI 8

*********************************************************
SOURCE OF
VARIATION                  SS           DF       MS        F
*********************************************************

BETWEEN                 36.02           1     36.02      .80
WITHIN               15444.01         341     45.29

     TOTAL           15480.03         342
*********************************************************
```

```
       UNIVARIATE ANOVA ON -- DAYS ABS

*********************************************************
SOURCE OF
VARIATION                  SS           DF       MS        F
*********************************************************

BETWEEN                137.31           1    137.31     2.75
WITHIN               15705.38         314     50.02

     TOTAL           15842.69         315
*********************************************************
```

```
     UNIVARIATE ANOVA ON -- UNITS CVRD

************************************************************
SOURCE OF
VARIATION                  SS           DF        MS            F
************************************************************

BETWEEN                 612.60            1    612.60        72.63
WITHIN                 2876.28          341      8.43

     TOTAL             3488.89          342
************************************************************
```

APPENDIX 8

HOMOGENEITY OF REGRESSION

```
DEPENDENT VARIABLE -- POST COMP

************************************************************


RAW SCORE REGRESSION WEIGHTS

           UNITS    STUDEN   PRE CO   PRE AP   DAYS A      RSQ          R         F

 GROUP 1    .594     .027     .723     .259    -.206      .568       .754    56.367
 GROUP 2   1.362     .395     .511     .166    -.126      .448       .669    14.594

  POOLED    .854     .150     .647     .229    -.187      .506       .712    63.625
  TOTAL    1.006     .152     .642     .226    -.203      .520       .721    67.109

************************************************************


 TEST OF THE HYPOTHESIS OF HOMOGENEITY OF REGRESSION

          F =    3.243 WITH 5 AND 304 DEGREES OF FREEDOM.
```

```
DEPENDENT VARIABLE -- POST APPL

************************************************************


RAW SCORE REGRESSION WEIGHTS

             UNITS    STUDEN   PRE CO   PRE AP   DAYS A       RSQ          R          F

  GROUP 1    -.068     .159     .179     .636     .077        .559        .748     54.232
  GROUP 2    1.013     .141     .226     .755    -.210        .596        .772     26.537

   POOLED     .260     .149     .178     .652     .017        .553        .744     76.701
   TOTAL      .350     .151     .175     .650     .008        .556        .745     77.492

************************************************************


 TEST OF THE HYPOTHESIS OF HOMOGENEITY OF REGRESSION

          F =    2.478 WITH 5 AND 304 DEGREES OF FREEDOM.
```

APPENDIX 9

ANALYSIS OF COVARIANCE -- SMSG SCALES

```
DEPENDENT VARIABLE -- POS-SMSG 1

***************************************************************


RAW SCORE REGRESSION WEIGHTS
                  UNITS   STUDEN   DAYS A   PRE-SM   PRE-SM   PRE-SM   PRE-SM

 GROUP 1          .060     .023    -.009     .362     .012     .107     .027
 GROUP 2          .075     .005    -.002     .435     .035    -.030     .167

  POOLED          .069     .016    -.008     .381     .033     .064     .066
  TOTAL           .001     .016    -.001     .360     .042     .058     .085

***************************************************************



                 PRE-SM      RSQ         R          F

                 -.043      .271       .521      9.634
                 -.180      .256       .506      3.744

                 -.087      .256       .506     13.031
                 -.072      .215       .464     10.383


   TEST OF THE HYPOTHESIS OF HOMOGENEITY OF REGRESSION

           F =      .503 WITH 8 AND 294 DEGREES OF FREEDOM.
```

```
                  ANALYSIS OF COVARIANCE

   ************************************************************

   SOURCE OF VARIATION    ADJ. SS      DF       ADJ. MS      F

   ************************************************************

   REGRESSION             178.384       8.      22.298    11.312

   TREATMENT MEANS         63.225       1.      63.225    32.075
   HETEROGENEITY
     OF REGRESSION          7.934       8.        .992      .503

        ERROR             579.531     294.       1.971

             TOTAL        829.074     311.



DEPENDENT VARIABLE -- POS-SMSG 2

************************************************************


RAW SCORE REGRESSION WEIGHTS

             UNITS    STUDEN   DAYS A   PRE-SM   PRE-SM   PRE-SM   PRE-SM

  GROUP 1    .103     .016     -.034     .064     .206     .188     .082
  GROUP 2    .163     .012     -.060    -.284     .493     .251     .127

   POOLED    .126     .011     -.036    -.014     .295     .191     .098
   TOTAL     .062     .011     -.030    -.034     .303     .185     .115

************************************************************

             PRE-SM      RSQ         R          F

             -.255      .189       .435      6.046
             -.045      .264       .514      3.909

             -.172      .197       .444      9.309
             -.158      .168       .410      7.670
   TEST OF THE HYPOTHESIS OF HOMOGENEITY OF REGRESSION

            F =     .991 WITH 8 AND 294 DEGREES OF FREEDOM.



                 ANALYSIS OF COVARIANCE

**************************************************************

SOURCE OF VARIATION    ADJ. SS      DF       ADJ. MS      F

**************************************************************

REGRESSION             218.991        8.     27.374     8.058

TREATMENT MEANS         55.732        1.     55.732    16.405
HETEROGENEITY
  OF REGRESSION         26.921        8.      3.365      .991

     ERROR             998.805      294.      3.397

           TOTAL      1300.449      311.
```

```
DEPENDENT VARIABLE -- POS-SMSG 3

***********************************************************


RAW SCORE REGRESSION WEIGHTS

           UNITS    STUDEN   DAYS A   PRE-SM   PRE-SM   PRE-SM   PRE-SM

 GROUP 1   .066     .022    -.014     .121     .124     .450     .088
 GROUP 2   .191     .030    -.027     .251    -.063     .248     .076

  POOLED   .113     .026    -.019     .154     .067     .365     .073
  TOTAL    .103     .026    -.018     .151     .068     .364     .076

***********************************************************

            PRE-SM       RSQ         R          F

             .086        .309       .556     11.569
            -.218        .209       .458      2.879

            -.014        .250       .500     12.650
            -.012        .253       .503     12.800

TEST OF THE HYPOTHESIS OF HOMOGENEITY OF REGRESSION

         F =    1.307 WITH 8 AND 294 DEGREES OF FREEDOM.


                    ANALYSIS OF COVARIANCE

  ***********************************************************

  SOURCE OF VARIATION   ADJ. SS     DF      ADJ. MS     F

  ***********************************************************

  REGRESSION            261.144      8.     32.643    12.885

  TREATMENT MEANS         1.411      1.      1.411      .557
  HETEROGENEITY
    OF REGRESSION        26.483      8.      3.310     1.307

       ERROR            744.804    294.      2.533

            TOTAL      1033.843    311.
```

```
DEPENDENT VARIABLE -- POS-SMSG 4

**************************************************************


RAW SCORE REGRESSION WEIGHTS

                UNITS    STUDEN   DAYS A   PRE-SM   PRE-SM   PRE-SM   PRE-SM

 GROUP 1        .058     .027     -.017     .280     .031     .092     .347
 GROUP 2        .165     .035     -.094    -.016     .218     .167     .300

  POOLED        .093     .026     -.034     .236     .081     .085     .326
  TOTAL         .052     .026     -.030     .223     .087     .081     .337

**************************************************************


                PRE-SM     RSQ         R         F

                -.040      .288       .537    10.477
                -.238      .239       .489     3.422

                -.092      .249       .499    12.589
                -.082      .236       .486    11.730



  TEST OF THE HYPOTHESIS OF HOMOGENEITY OF REGRESSION

            F =    1.019 WITH 8 AND 294 DEGREES OF FREEDOM.



                     ANALYSIS OF COVARIANCE

**************************************************************

SOURCE OF VARIATION   ADJ. SS      DF      ADJ. MS       F

**************************************************************

REGRESSION            362.509       8.     45.314     11.936

TREATMENT MEANS        23.444       1.     23.444      6.176
HETEROGENEITY
  OF REGRESSION        30.945       8.      3.868      1.019

     ERROR           1116.099     294.      3.796

         TOTAL       1532.997     311.
```

```
DEPENDENT VARIABLE -- POS-SMSG 5

**************************************************************

RAW SCORE REGRESSION WEIGHTS

            UNITS    STUDEN   DAYS A   PRE-SM   PRE-SM   PRE-SM   PRE-SM

GROUP 1      .012     .004    -.011     .117     .015     .116     .086
GROUP 2      .085     .015    -.018    -.027     .012     .137    -.042

 POOLED      .041     .007    -.011     .081     .004     .107     .047
 TOTAL       .035     .007    -.011     .080     .005     .106     .049

**************************************************************

            PRE-SM       RSQ          R           F

             .144        .227        .477       7.616
             .153        .130        .361       1.630

             .155        .160        .400       7.198
             .156        .159        .398       7.149


  TEST OF THE HYPOTHESIS OF HOMOGENEITY OF REGRESSION

            F =     1.645 WITH 8 AND 294 DEGREES OF FREEDOM.


                ANALYSIS OF COVARIANCE

**************************************************************

SOURCE OF VARIATION   ADJ. SS      DF       ADJ. MS      F

**************************************************************

REGRESSION             43.870       8.       5.484     7.263

TREATMENT MEANS          .494       1.        .494      .655
HETEROGENEITY
  OF REGRESSION         9.935       8.       1.242     1.645

    ERROR             221.979     294.        .755

         TOTAL        276.279     311.
```

APPENDIX 10

STEPWISE REGRESSION

1. Dependent Variable -- Post Comp.

Independent Variables	Mult. R	Mult. R^2	Increase in R^2
Pre Comp.	.67	.45	.45
Pre SMSG 4	.72	.51	.06
Pre Appl.	.73	.53	.02
Days Abs.	.74	.54	.01
Pre SMSG 1	.74	.55	.01
Pre Att 6	.75	.56	.01

2. Dependent Variable -- Post Appl.

Independent Variables	Mult. R	Mult. R^2	Increase in R^2
Pre Appl.	.72	.51	.51
Reading	.74	.54	.03
Pre Att 2	.75	.56	.02
Pre SMSG 4	.76	.57	.01

3. Dependent Variable -- Post SMSG 1

Independent Variables	Mult. R	Mult. R^2	Increase in R^2
Pre Appl.	.43	.18	.18
Pre SMSG 1	.50	.25	.07
Reading	.53	.28	.03
Pre Att 4	.54	.29	.01
Pre Att 7	.56	.31	.02

4. Dependent Variable -- Post SMSG 2

Independent Variables	Mult. R	Mult. R^2	Increase in R^2
Pre SMSG 4	.30	.09	.09
Pre SMSG 2	.36	.13	.04
Pre SMSG 3	.40	.16	.03
I Q	.43	.18	.02

5. Rep. Variable -- Post SMSG 4

Independent Variables	Mult. R	Mult. R^2	Increase in R^2
Pre SMSG 4	.48	.23	.23
Reading	.54	.29	.06
Pre SMSG 1	.56	.32	.03

APPENDIX 11

SAT Scale Score Gains -- Absentees vs. Non-absentees

Experimental Group

	Non-absentees N = 210	Absentees N = .30	Difference	Direction
Post-Computation	5.79	5.32		
Pre-Computation	4.66	4.11		
Gain	1.13	1.21	0.08	A > N
Post-Applications	5.86	6.00		
Pre-Applications	5.42	5.57		
Gain	0.44	0.33	0.11	A < N

Control Group

	Non-absentees N = 110	Absentees N = 12	Difference	Direction
Post-Computation	6.49	6.63		
Pre-Computation	4.68	4.55		
Gain	1.81	2.08	.27	A > N
Post-Applications	6.29	6.20		
Pre-Applications	5.64	5.74		
Gain	0.65	0.46	.19	A < N